Smarter Balanced Assessment Consortium:
Mathematics Item Specifications
High School

Developed by Measured Progress/ETS Collaborative
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Smarter Balanced Mathematics Item Specifications High School
The SMARTER Balanced Assessment Consortium (Smarter Balanced) is committed to using evidence-centered design (ECD) in its development of its assessment system. As part of this design, the Smarter Balanced established four “Claims” regarding what students should know and be able to do in the domain of mathematics. The principles of ECD require that each of these four claims be accompanied by statements about the kinds of evidence that would be sufficient to support the claims. These evidence statements are articulated as “assessment targets.”

The initial work of using ECD to establish both the four claims and the accompanying targets for mathematics has been completed by the consortium and is articulated in the document “Content Specifications for the Summative Assessment of the Common Core State Standards for Mathematics (CCSSM).” The content specifications document is a necessary complement to these test item and performance task specifications, and the documents should be used together to fully understand how to write the items and tasks needed to provide the evidence to support these claims and targets.

What follows is a guide to help item writers develop items/tasks that meet the requirements set forth by the Smarter Balanced. This specification document will also provide useful information to other stakeholders concerning the types of items/tasks that will comprise the summative assessments beginning in 2015.
Assessing Mathematics

The Common Core State Standards for Mathematics require that mathematical content and mathematical practices be connected (CCSSM, p. 8). In addition, two of the major design principles of the standards are focus and coherence (CCSSM, p. 3). Together, these features of the standards have important implications for the design of the Smarter Balanced assessment system.

The next section will discuss the various types of items/tasks the Smarter Balanced intends to use in its assessment system in order to connect the mathematics content with the mathematical practices defined in the CCSS. The following definitions are used for various parts of an item or task.

- **Item**: the entire item, including the stimulus, question/prompt, answer/options, scoring criteria, and metadata.
- **Task**: similar to an item, yet typically more involved and usually associated with constructed-response, extended-response, and performance tasks.
- **Stimulus**: the text, source (e.g., video clip), and/or graphic about which the item is written. The stimulus provides the context of the item/task to which the student must respond.
- **Stem**: the statement of the question or prompt to which the student responds.
- **Options**: the responses to a selected-response (SR) item from which the student selects one or more answers.
- **Distractors**: the incorrect response options to an SR item.
- **Distractor Analysis**: the item writer’s analysis of the options or rationale for inclusion of specific options.
- **Key**: the correct response(s) to an item.
- **Top-Score Response**: one example of a complete and correct response to an item/task.
- **Scoring Rubric**: the descriptions for each score point for an item/task that is assigned more than one point for a correct response.

**Stimulus for Mathematics Items/Tasks**

Stimulus materials for mathematics items/tasks usually take the form of graphs, models, figures, etc. that students must read and examine in order to respond to the items/tasks. Below are some general guidelines for mathematics stimulus materials. For a comprehensive discussion of stimulus materials, refer to the specific section of the Test Item and Performance Task Specifications that is devoted entirely to Stimulus Specifications.

- Graphs, pictures, models, tables, figures, and any other graphic presented with an item or task must meet the art and style guidelines adopted by the Smarter Balanced. When using graphics,
• Use contexts that are both familiar and meaningful to the mathematics being assessed, as well as appropriate to the grade level of the test takers.

• Keep the amount of reading to a minimum and non-mathematics vocabulary should be at least one grade level below that of the test takers.

• Include reliable source and verification information with the item/task when referencing factual information and using real-world data.

• Use visuals that mirror and parallel the wording and expectations of the accompanying text;

• Illustrate only necessary information in the graphics so as not to confuse and/or distract test takers; and

• Represent important parts of the item/task in visual images if the graphics serve to increase item access for more test takers.
Selected-Response Items

Traditionally, selected-response (SR) items include a stimulus and stem followed by three to five options from which a student is directed to choose only one or best answer. By redesigning some SR items, it is often possible to both increase the complexity of the item and yield more useful information regarding the level of understanding about the mathematics that a student’s response demonstrates. For example, consider the following SR item in which one of the four options is the correct response (Figure 1).

Figure 1.

Which model below best represents the fraction $\frac{2}{5}$?

A.  

B.  

C.  

D.  

Even if a student does not truly have a deep understanding of what 2/5 means, he or she is likely to choose option B over the rest of the options because it looks to be a more traditional way of representing fractions. By a simple restructuring of this problem into a multi-part item, including a modification to option C, a clearer sense of how deeply a student understands the concept of 2/5 can be ascertained (see Figure 2).
This item is more complex in that a student now has to look at each part separately and decide whether 2/5 can take different forms. By assigning two points to this problem, we can also provide feedback at the item level as to the depth of understanding a student has about simple fractions. The total number of ways to respond to this item is 16. “Guessing” the correct combination of responses is much less likely than for a traditional 4-option selected-response item. The correct response for this item will receive 2 points, and the points will be earned based on the level of understanding the student has demonstrated. A suggested scoring rubric for this item follows in Figure 3.
Scoring Rubric

Responses to this item will receive 0-2 points, based upon the following:

2 points: YNYN  The student has a solid understanding of 2/5 as well as an equivalent form of 2/5.

1 point:  YNNN, YYNN, YYYN   The student has only a basic understanding of 2/5. Either the student doesn’t recognize an equivalent fraction for 2/5 or doesn’t understand that all 5 parts must be equal-sized in figure 1b.

0 points:  YYYY, YNNY, NNNN, NNYY, NYYN, NYYN, NYYY, NYNN, NNNN, NYNY, NNYN, NNNY   The student demonstrates inconsistent understanding of 2/5 or answers “Y” to figure 1d, clearly showing a misunderstanding of what 2/5 means. Figure 1d is considered a “disqualifier” and an answer of “Y” to this part of the item would cancel out any other correct responses as “guesses” on the part of the student.

Other ways to format SR items include providing options in which more than one choice is correct (e.g., 3 out of 8 options), multiple-part yes/no questions, and even multiple-part items that require students to match descriptions of a term or activity to the corresponding option.

It is important to note that a writer should only consider some of these alternate SR formats when, by doing so, they serve to provide more information about a student’s knowledge, skills, and ability (KSA) than a simpler 4- or 5-option SR item would demonstrate.
General Guidelines for Developing SR Items

- Each item should be written to focus primarily on one assessment target. Secondary targets are acceptable and should be listed in the item forms as appropriate, but it should be clear to all stakeholders which assessment target is the focal point of the item.

- Items should be appropriate for students in terms of grade-level difficulty, cognitive complexity, and reading level.¹

- Items should not provide an advantage or disadvantage to a particular group of students. Items should not exhibit or reflect disrespect toward any segment of the population with regard to age, gender, race, ethnicity, language, religion, socioeconomic status, disability, or geographic region.²

- Items are expected to include mathematical concepts detailed in the CCSSM of lower grades.

- At high school, most items should be written so they can be answered without using a calculator.³ However, some targets may require the use of an online calculator tool in order to efficiently problem solve. In these cases, the calculator tool will appear in the specification table under “allowable tools.”

- Items should provide clear and complete instructions to students.

- Each item should be written to clearly elicit the desired evidence of a student’s KSA.

- Options should be arranged according to a logical order (e.g., alphabetical, least to greatest value, greatest to least value, length of options).

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¹ For mathematics items, the reading level should be approximately one grade level below the grade level of the test, except for specifically assessed mathematical terms or concepts.

² This guideline is discussed in greater detail in the Bias/Sensitivity and Accessibility portions of the Specifications.

³ Graphing and scientific calculators may be used for many items in high school mathematics assessments, even if unnecessary to solve the problem. An online version will be available for most items during the CAT portion of the assessment, except when specifically “turned off” because of the particular content of the item being assessed.
Constructed-Response Items

The main purpose of a constructed-response (CR) item/task is to address targets and claims that are of greater complexity, requiring more analytical thinking and reasoning than an SR item can typically elicit. Additionally, fill-in-the-blank type CR items (CRs) can markedly increase the discrimination factor and reliability of comparable SR items (SRs) by virtually eliminating the “guessing” element of those items.

The Smarter Balanced mathematics assessments will include a wide variety of CRs. A primary distinction that will be made in the development of CR items/tasks will be related to the portion of the assessment to which the item/task is assigned. As noted earlier in this document, the design of the assessment includes both a computer-adaptive component and a performance task component. It is desirable to have CRs that cover a variety of content domains in both components. Out of necessity, only the CRs that can be computer scored using current technologies, including artificial intelligence (AI), will be assigned to the computer-adaptive component of the assessment. All other CRs will be assigned to a collection of 6 to 9 tasks that are intended to collectively take up to 120 minutes to administer.

In order to distinguish the CR items/tasks that contribute to the performance task component from those that are part of the computer-adaptive component, the former will be referred to as extended-response (ER) items/tasks. Therefore, a CR designation in mathematics will refer to brief constructed-response items that focus on a particular skill or concept. These items are intended to be part of the computer-adaptive component. The length of time these CRs take to administer should typically vary from 1 to 5 minutes. However, there is room for development of highly-scaffolded CR tasks that may take up to 10 minutes to complete, yet will still be part of the computer-adaptive component of the assessment.

As stated above, an ER item/task will be administered during the performance task component of the assessment. It is intended that no single ER be administered in isolation, but rather as part of a collection of 6 to 9 ER items/tasks that will serve to complete the distribution of content and targets for a well-designed assessment, appropriate to each grade.

As stated in the content specifications,

“...for much school-level mathematics, paper and pencil remains the natural medium for working mathematically, as it allows for diverse representations such as quick sketches of diagrams or graphs, and for mathematical expressions and tables to be rapidly created and freely mixed. Doing similar exploratory work on a computer would require the time-consuming use of multiple, specialized tools, which were often designed for producing polished presentations or setting up large-scale computations rather than as a ‘scratchpad’ for mathematical thinking.”
When the work itself is important to assess, a carefully crafted ER item/task can often provide the best means to evaluate students’ work. Extended-response items/tasks also afford the opportunity to assess substantial chains of reasoning as the standards require.

The time allotted to administer ER items/tasks should vary from 5 to 20 minutes, depending on whether the item/task is written to measure targets associated with Claim 2, Claim 3, or Claim 4.

### General Guidelines for Developing CR and ER Items/Tasks

- Each item/task should be written to assess a primary content domain as identified in the assessment targets for Claim 1 of the specified grade. Secondary content domains are also possible and should be listed in order of prominence when completing the item form.

- Items/tasks should be appropriate for students in terms of grade-level difficulty, cognitive complexity, and reading level.\(^4\)

- Items/tasks should not provide an advantage or disadvantage to a particular group of students. Items should not exhibit or reflect disrespect toward any segment of the population with regard to age, gender, race, ethnicity, language, religion, socioeconomic status, disability, or geographic region.\(^5\)

- Items/tasks are expected to include mathematical concepts detailed in the CCSS of lower grades.

- At high school, most items should be written so they can be answered without using a calculator.\(^6\) However, some targets may require the use of an online calculator tool in order to efficiently problem solve. In these cases, the calculator tool will appear in the specification table under “allowable tools.”

- Items/tasks should provide clear and complete instructions to students.

- CR items/tasks should be written to clearly elicit the desired evidence of a student’s KSA.

- For CR items, a complete key and/or scoring rubric must be included with the item along with a justification for the solution, as needed.

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\(^4\) For mathematics items, the reading level should be approximately one grade level below the grade level of the test, except for specifically assessed mathematical terms or concepts.

\(^5\) This guideline is discussed in greater detail in the Bias/Sensitivity and Accessibility portions of the Specifications.

\(^6\) Graphing and scientific calculators may be used for many items in high school mathematics assessments, even if unnecessary to solve the problem. An online version will be available for most items during the CAT portion of the assessment, except when specifically “turned off” because of the particular content of the item being assessed.
For ER items/tasks, a “Sample Top-Score Response” must be included, accompanied by a scoring rubric that details the rationale for awarding each score point in terms of the evidence demonstrated by a student’s response.

Technology-Enhanced Items/Tasks

Technology-enhanced (TE) items/tasks are desirable when they can provide evidence for mathematical practices that could not be as reliably obtained from SR and CR items. Additionally, components of certain extended-response (ER) items and performance tasks may employ TE tools as part of the task. An expressed desire on the part of the consortium is that the use of TE items in the assessments will ultimately encourage classroom use of authentic mathematical computing tools (e.g., spreadsheets, interactive geometry software) as part of classroom instruction.

A specific section of the Test Item and Performance Task Specifications is devoted entirely to the development of TE items. Determining whether a TE item/task is the best vehicle with which to assess a particular piece of evidence will require a careful analysis of the TE section of these Specifications.

Performance Tasks

As with TE items/tasks, an entire section of these Specifications contains information related to the development of high-quality performance tasks, and a writer must refer to that section when attempting to write these tasks. In short, performance tasks should:

- Integrate knowledge and skills across multiple claims and targets.
- Measure capacities such as depth of understanding, research skills, and/or complex analysis with relevant evidence.
- Require student-initiated planning, management of information/data and ideas, and/or interaction with other materials.
- Reflect a real-world task and/or scenario-based problem.
- Allow for multiple approaches.
- Represent content that is relevant and meaningful to students.
- Allow for demonstration of important knowledge and skills, including those that address 21st century skills such as critically analyzing and synthesizing information presented in a variety of formats, media, etc.
- Require scoring that focuses on the essence of the Claim(s) and Targets for which the task was written.
- Be feasible for the school/classroom environment.

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7 Scoring guidelines for ERs, as well as PTs, are discussed more thoroughly in the sections of this document that immediately follow the Specification Tables for Claims 2, 3, and 4.

8 Scoring rules are described in detail in the Performance Task section of the Specifications.
Many PTs will require up to 120 minutes in which to administer. Additional time might be necessary for prework or group work, as required by a particular task.

The table below represents the general structure of most mathematics PTs.

<table>
<thead>
<tr>
<th>Mathematics Performance Task</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Stimulus</strong></td>
</tr>
<tr>
<td>• graphs</td>
</tr>
<tr>
<td>• video clips</td>
</tr>
<tr>
<td>• maps</td>
</tr>
<tr>
<td>• photos</td>
</tr>
<tr>
<td>• research reports</td>
</tr>
<tr>
<td>• geometric figures</td>
</tr>
<tr>
<td>• 2-D and 3-D models</td>
</tr>
<tr>
<td>• spreadsheets</td>
</tr>
<tr>
<td>• areas of math content (algebra, geometry.), etc.</td>
</tr>
</tbody>
</table>
Computational Complexity of Mathematics Items and Tasks

In general, items/tasks developed to assess student understanding of core concepts and procedures in mathematics will draw upon grade-level standards to ensure student mastery of this content. Grade-level standards are implicitly included in the assessment targets of Claim 1 for high school. The assessment targets for Claim 1 correspond directly to the cluster headings contained in the CCSSM, and the technical demand of these Claim 1 items can be consistent with the grade level being assessed. However, when writing more complex tasks (such as those associated with Claims 3 and 4), the computational demand should be lowered and should typically be met by content that was first taught in earlier grades.

Use of Formulas

For many items/tasks, students will be expected to know the formulas that are used to solve problems (e.g., formulas for area, perimeter, volume, etc.). A reference sheet of formulas will not be available to students for these items. However, for more complex problems, a search function may be available (on an item-by-item basis) that allows a student to call up a table of formulas. This table will not include names of the formulas or pictures of the figures for which the formulas are relevant, and will serve only to prompt a student’s memory of the correct formula. For some TE items, it is possible that drop-down menus will be available for students to choose a relevant formula for a particular item.
Claim 1: Concepts and Procedures

Claim 1 — Students can explain and apply mathematical concepts and interpret and carry out mathematical procedures with precision and fluency.

Rationale for Claim 1

This claim addresses procedural skills and the conceptual understanding on which developing skills depend. It is important to assess how aware students are of how concepts link together and why mathematical procedures work the way they do. Central to understanding this claim is making the connection to these elements of the mathematical practices as stated in the CCSSM:

Use appropriate tools strategically.

- Use technological tools to explore and deepen their understanding of concepts.

Attend to precision.

- State the meaning of the symbols they choose, including using the equal sign consistently and appropriately.
- Specify units of measure and label axes to clarify the correspondence with quantities in a problem.
- Calculate accurately and efficiently, and express numerical answers with a degree of precision appropriate for the problem context.
  - Older students should be able to examine claims and make explicit use of definitions.

Look for and make use of structure.

- Look closely to discern a pattern or structure.
  - Young students might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have.
  - Later, students will see $7 \times 8$ equals the well remembered $7 \times 5 + 7 \times 3$, in preparation for the distributive property.
  - In the expression $x^2 + 9x + 14$, older students can see the $14$ as $2 \times 7$ and the $9$ as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems.
- See complicated things, such as some algebraic expressions, as single objects or composed of several objects.

Look for and express regularity in repeated reasoning.

- Notice if calculations are repeated.
• Look for both general methods and shortcuts.
  o Middle school students might abstract the equation \((y-2)/(x-1)=3\) by paying attention to the calculation of slope as they repeatedly check whether the points are on the line through \((1, 2)\) with a slope 3.
  o Noticing the regularity in the way terms cancel when expanding \((x-1)(x+1)(x^2+1)\) and \((x-1)(x^3+x^2+x+1)\) might lead high school students to the general formula for the sum of a geometric series.
• Maintain oversight of the process of solving a problem while attending to the details.
• Continually evaluate the reasonableness of intermediate results.

**Essential Properties of Items/Tasks that Assess Claim 1**

Items/tasks assessing this claim will include SR items, TE items/tasks, and short CR items/tasks that focus on a particular skill or concept. They will also include items that require students to translate between representations of concepts (words, diagrams, symbols) and items that require the identification of structure.

It is important to note that Claim 1 specification tables are the only ones in which a direct connection to the content domains and clusters of the grade-level CCSSM is made. Items/tasks designed to elicit the evidence sought in Claims 2, 3, and 4 will necessarily rely on the content explicated in the Claim 1 specification tables.

Specification tables have been developed for each assessment target associated with Claim 1. These tables are intended to provide guidance to item writers for the development of items/tasks that primarily assess Claim 1. Figure 4 provides the model used for all Claim 1 tables along with an explanation of the metadata that populates each cell of the table.
**Figure 4. Sample Table**

<table>
<thead>
<tr>
<th><strong>Claim 1:</strong> Concepts and Procedures</th>
<th>Students can explain and apply mathematical concepts and interpret and carry out mathematical procedures with precision and fluency.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Content Domain:</strong> This cell contains the content domain associated with the specified target. For high school, eligible domains are: Number and Quantities: The Real Number System, Quantities, Algebra: Seeing Structure in Expressions, Arithmetic with Polynomials and Rational Expressions, Creating Equations, Reasoning with Equations and Inequalities, Functions: Building Functions, Geometry: Congruence, and Statistics and Probability: Interpreting Categorical and Quantitative Data.</td>
<td></td>
</tr>
<tr>
<td><strong>Target ____ [ ]:</strong> Assessment Target letter and (emphasis), as defined by the Content Specifications. The emphasis designation is identified by “m” for major, “s” for supporting, and “a” for additional. The complete text of the target will populate the rest of this cell.</td>
<td></td>
</tr>
<tr>
<td><strong>Standards:</strong> Standards from the CCSSM related to the specified target.</td>
<td></td>
</tr>
<tr>
<td><strong>DOK Target(s):</strong> Depth-of-Knowledge level(s) assigned to the specified target.</td>
<td></td>
</tr>
<tr>
<td><strong>Evidence Required:</strong> Statements that define the knowledge, skills, or abilities a student must demonstrate in order to provide evidence in support of one or more aspects of the target and claim.</td>
<td></td>
</tr>
<tr>
<td><strong>Allowable Item Types</strong>: The item types allowed for this target (SR, CR, or TE).</td>
<td></td>
</tr>
<tr>
<td><strong>Task Models:</strong> A task model describes key characteristics or features that items are to have in order to establish a context or problem that elicits the desired evidence from the student. In effect, a task model describes what the prompt is intended to ask of the student, the content or materials (stimuli) that the student is supposed to work with when applying the targeted knowledge, skill, or ability, and any unique interactions that the item must support in order to allow the student to produce the desired response information. For every enumerated statement of “evidence required,” a corresponding task model will follow. If more than one type of item/task is appropriate for the same evidence statement, then the same number will be assigned. The variables will be the item type and DOK level associated with the task model.</td>
<td></td>
</tr>
<tr>
<td><strong>Allowable Stimulus Materials:</strong> This cell lists the kinds of stimuli that can be used. It is not to be considered a complete list, but suggests various types.</td>
<td></td>
</tr>
<tr>
<td><strong>Allowable Disciplinary Vocabulary:</strong> This cell suggests mathematics-specific vocabulary that students are expected know, as related to the target.</td>
<td></td>
</tr>
<tr>
<td><strong>Allowable Tools:</strong> This cell identifies allowable tools that students may use in working with the item/task (e.g., calculator, protractor, etc.).</td>
<td></td>
</tr>
<tr>
<td><strong>Target-Specific Attributes:</strong> This cell identifies specific attributes, related to the target, which could include limitations on the content or other considerations.</td>
<td></td>
</tr>
<tr>
<td><strong>Key Nontargeted Constructs:</strong> This cell identifies knowledge and skills the student needs in order to respond, but which are not scored for the specified target.</td>
<td></td>
</tr>
<tr>
<td>Accessibility Concerns:</td>
<td>This cell identifies possible concerns for students with disabilities or those with other accessibility issues.</td>
</tr>
<tr>
<td>------------------------</td>
<td>----------------------------------------------------------------------------------------------------------</td>
</tr>
<tr>
<td>Sample Items:</td>
<td>This cell contains item codes that are &quot;hot-button&quot; links with access to sample items for the specified target.</td>
</tr>
</tbody>
</table>

*SR = selected-response item; CR = constructed-response item; TE = technology-enhanced item; ER = extended-response item; PT = performance task
How to Complete the Item Form for Claim 1 Targets

Items that are written to Claim 1 assessment targets must follow the guidelines contained throughout these specifications. Additionally, item writers must complete an Item Form for every submitted item. Figure 5 provides the model used for all Claim 1 item forms along with an explanation of the metadata that populates each cell of the form.

**Figure 5.**

<table>
<thead>
<tr>
<th>Sample Item ID:</th>
<th>MAT.GR.IT.1.CDOMA.T.xxx (see below)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Grade:</td>
<td>Specify the 2-digit grade level (HS for high school).</td>
</tr>
<tr>
<td>Claim(s):</td>
<td>Enter the number and text of the primary <em>Smarter Balanced</em> claim. If more than one claim is part of the item/task, the first number must represent the <strong>primary claim</strong>, with secondary and tertiary claims listed by order of importance.</td>
</tr>
<tr>
<td>Assessment Target(s):</td>
<td>Enter the <em>Smarter Balanced</em> target alpha character(s) and the text of the <em>Smarter Balanced</em> target(s).</td>
</tr>
<tr>
<td>Content Domain:</td>
<td>Enter the primary CCSSM domain associated with the claim and target.</td>
</tr>
<tr>
<td>Standard(s):</td>
<td>Enter the number(s) of the CCSSM standard(s).</td>
</tr>
<tr>
<td>Mathematical Practice(s):</td>
<td>Specify the mathematical practices (1-8) associated with the item/task.</td>
</tr>
<tr>
<td>DOK:</td>
<td>Specify the Depth of Knowledge level (1-4) of the item/task.</td>
</tr>
<tr>
<td>Item Type:</td>
<td>Specify the item type (SR, CR, TE).</td>
</tr>
<tr>
<td>Score Points:</td>
<td>Specify the total point value of the item.</td>
</tr>
<tr>
<td>Difficulty:</td>
<td>Specify the estimated difficulty of item (L=Low, M=Medium, H=Hard). See below for further explanation of coding.</td>
</tr>
<tr>
<td>Key:</td>
<td>Specify the correct key for SR items or indicate “See Sample Top-Score Response” for multi-point items/tasks.</td>
</tr>
<tr>
<td>Stimulus/Source:</td>
<td>Specify any stimulus material used and/or source required for factual information. All sources must be reliable and reproducible. If none, leave blank.</td>
</tr>
<tr>
<td>Target-Specific Attributes (e.g., accessibility issues):</td>
<td>Specify any target-specific attributes (e.g., accessibility issues).</td>
</tr>
<tr>
<td>Notes:</td>
<td>Add any notes here that you believe will aid in understanding the purpose of this sample item. For TE items, include the TE template name here.</td>
</tr>
</tbody>
</table>

**Sample Item ID:** Specify the sample item ID “MAT.GR.IT.C.CDOMA.T.xxx”

**MAT – Mathematics**

**GR – 03, 04, 05, 06, 07, 08, or HS**

**IT – Item type (SR, CR, TE, ER, or PT)**

**C – Claim number 1, 2, 3, or 4**

**CDOMA – Content Domain letters from CCSSM (e.g., OA, MD, must be five places, lead with zeros until all five places are filled)**

**Note:** For PTs, the content domain field is filled with the task name abbreviation.

**T – Primary Assessment target alpha (A, B, C, D, etc.)**

**xxx – Item number. Leave alone for now; will be assigned after acceptance.**

**Difficulty** – Base the level on the percent of students that would be expected to get the item/task correct or would earn the maximum number of points as follows:

- Low – greater than 70%
- Medium – 40% to 70%
- Hard – less than 40%

*These are all intended to be “hot-button” links to the complete text of each component.*
**Item/Task:**

Use this space for the stem, stimulus, and/or options. The font size for items/tasks is Verdana, 14-point.

<table>
<thead>
<tr>
<th><strong>Key and Distractor Analysis or Scoring Rubric for Multi-Part Items/Tasks:</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>The information in this section will vary, depending on the item type below.</td>
</tr>
<tr>
<td>For traditional 1-point SR items: Include the key in both the item form and in this section and provide complete rationales for each distractor.</td>
</tr>
<tr>
<td>For multi-point SR items: Include a rubric and justification for each score-point. Refer to the details presented in Figure 3.</td>
</tr>
<tr>
<td>For short CR items/tasks: Provide a sample top-score response, followed by a rubric and justification for each score point (see directions after Claim 2 Item Form).</td>
</tr>
<tr>
<td>For TE items/tasks: Provide a sample top-score response and description of the interaction(s) the student must demonstrate to score each point (may be only 1 point).</td>
</tr>
</tbody>
</table>
Claim 2: Problem Solving

Claim 2 — Students can solve a range of complex, well-posed problems in pure and applied mathematics, making productive use of knowledge and problem-solving strategies.

Claim 2 Overview

Assessment items and tasks focused on Claim 2 include problems in pure mathematics and problems set in context. Problems are presented as items and tasks that are well-posed (that is, problem formulation is not necessary) and for which a solution path is not immediately obvious.9 These problems require students to construct their own solution pathway rather than follow a provided one. Such problems will therefore be unstructured, and students will need to select appropriate conceptual and physical tools to use.

Essential Properties of Items/Tasks that Assess Claim 2

Claim 2 will be assessed using a combination of SR items, TE items, CR items/tasks, and ER items/tasks that focus on making sense of problems and using perseverance in solving them.

To preserve the focus and coherence of the standards as a whole, Claim 2 items/tasks must draw clearly on knowledge and skills that are articulated in the Smarter Balanced content standards. At each grade level, the content standards offer natural and productive settings for generating evidence for Claim 2. Items/tasks generating evidence for Claim 2 in a given grade may also draw upon knowledge and skills articulated in the progression of standards up to that grade.

The intent is that each of the targets should not lead to a separate item/task, but will provide evidence for several of the assessment targets defined for Claim 2. It is in using content from different areas, including work studied in earlier grades, that students demonstrate their problem-solving proficiency. For this reason, the specification tables will look somewhat different from the Claim 1 tables. Specifically, a separate table is not relevant at the target level, so all targets are included in a single, grade-level table for Claim 2. Another important distinction between Claim 1 specification tables (which link the content to other claims) is that the evidence required of students to satisfy Claim 2 centers around specific statements of the mathematical practices contained in the CCSSM. These statements are found in the cell labeled “Rationale,” again relying on the Content Specifications for clarity.

As stated above, Claim 1 specification tables are the only ones in which a direct connection to the content domains and clusters of the grade-level CCSSM is made. Therefore, items/tasks designed to elicit the evidence sought in Claim 2 will necessarily rely on the content explicated in the Claim 1 specification tables.

Figure 6 provides the model used for all Claim 2 tables. Most of the information contained in Figure 6 will be repeated in all Claim 2 tables for grades 6-8. Notes have been added to specific cells in order to clarify the information and/or source of the metadata contained in those cells.

---

Primary Claim 2: Problem Solving

Students can solve a range of well-posed problems in pure and applied mathematics, making productive use of knowledge and problem-solving strategies.

Secondary Claim(s): Items/tasks written primarily to assess Claim 2 will necessarily involve some Claim 1 content targets. Related Claim 1 targets should be listed below the Claim 2 targets in the item form. If Claim 3 or 4 targets are also directly related to the item/task, list those following the Claim 1 targets in order of prominence.

Primary Content Domain: Each item/task should be classified as having a primary, or dominant, content focus. The content should draw upon the knowledge and skills articulated in the progression of standards leading up to high school.

Secondary Content Domain(s): While tasks developed to assess Claim 2 will have a primary content focus, components of these tasks will likely produce enough evidence for other content domains that a separate listing of these content domains needs to be included where appropriate.

Assessment Targets: Any given item/task should provide evidence for several Claim 2 assessment targets. Each of the following targets should not lead to a separate task: it is in using content from different areas, including work studied in earlier grades, that students demonstrate their problem solving proficiency. Multiple targets should be listed in order of prominence as related to the item/task.

Target A: Apply mathematics to solve well-posed problems arising in everyday life, society, and the workplace. (DOK 2, 3)
Under Claim 2, the problems should be completely formulated, and students should be asked to find a solution path from among their readily available tools.

Target B: Select and use appropriate tools strategically. (DOK 1, 2)
Tasks used to assess this target should allow students to find and choose tools; for example, using a “Search” feature to call up a formula (as opposed to including the formula in the item stem) or using a protractor in physical space.

Target C: Interpret results in the context of a situation. (DOK 2)
Tasks used to assess this target should ask students to link their answer(s) back to the problem’s context. In early grades, this might include a judgment by the student of whether to express an answer to a division problem using a remainder or not based on the problem’s context. In later grades, this might include a rationalization for the domain of a function being limited to positive integers based on a problem’s context (e.g., understanding that the number of buses required for a given situation cannot be 32½, or that the negative values for the independent variable in a quadratic function modeling a basketball shot have no meaning in this context).

Target D: Identify important quantities in a practical situation and map their relationships (e.g., using diagrams, two-way tables, graphs, flowcharts, or formulas). (DOK 1, 2, 3)
For Claim 2 tasks, this may be a separate target of assessment explicitly asking students to use one or more potential mappings to understand the relationship between quantities. In some cases, item stems might suggest ways of mapping relationships to scaffold a problem for Claim 2 evidence.

Relevant Verbs: understand (often in conjunction with one or more other relevant
verbs), solve, apply, describe, illustrate, interpret, and analyze
[Note: This list of verbs came directly from the Content Specifications, immediately preceding the list of targets for Claim 2, "Relevant Verbs for Identifying Content Clusters and/or Standards for Claim 2."]

<table>
<thead>
<tr>
<th>DOK Target(s):</th>
<th>1, 2, 3 [Note: These are hot-button links to the full text.]</th>
</tr>
</thead>
</table>
| Claim 2 Rationale: | **Mathematical Practice 1: Make sense of problems and persevere in solving them.**
Mathematically proficient students:
• explain to themselves the meaning of a problem and look for entry points to its solution.
• analyze givens, constraints, relationships, and goals.
• make conjectures about the form and meaning of the solution attempt.
• plan a solution pathway rather than simply jumping into a solution.
• consider analogous problems and try special cases and simpler forms of insight into the solutions.
• monitor and evaluate their progress and change course if necessary.
• transform algebraic expressions or change the viewing window on their graphing calculator to get information.
• explain correspondences between equations, verbal descriptions, tables, and graphs.
• draw diagrams of important features and relationships, graph data, and search for regularity or trends.
• use concrete objects or pictures to help conceptualize and solve a problem.
• check their answers to problems using a different method.
• ask themselves, “Does this make sense?”
• understand the approaches of others in solving complex problems and identify correspondences between approaches.

**Mathematical Practice 5: Use appropriate tools strategically.**
Mathematically proficient students:
• consider available tools when solving a mathematical problem.
  (Tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software.)
• are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations.
• detect possible errors by using estimations and other mathematical knowledge.

**Mathematical Practice 7: Look for and make use of structure.**
Mathematically proficient students:
• look closely to discern a pattern or structure.
  o Young students might notice that three and seven more is the same amount as seven and three more or they may sort a collection of shapes according to how many sides the shapes
have.
  o Later, students will see $7 \times 8$ equals the well-remembered $7 \times 5 + 7 \times 3$, in preparation for the distributive property.
  o In the expression $x^2 + 9x + 14$, older students can see the 14 as $2 \times 7$ and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems.
  • step back for an overview and can shift perspective.
  • see complicated things, such as some algebraic expressions, as single objects or composed of several objects.

Mathematical Practice 8: Look for and express regularity in repeated reasoning.
Mathematically proficient students:
  • notice if calculations are repeated.
  • look for both general methods and shortcuts.
    o Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations and conclude they have a repeated decimal.
    o Middle school students might abstract the equation $(y-2)/(x-1)=3$ by paying attention to the calculation of slope as they repeatedly check whether the points are on the line through $(1, 2)$ with a slope 3.
  • maintain oversight of the process of solving a problem, while attending to the details.
  • continually evaluate the reasonableness of intermediate results.

<table>
<thead>
<tr>
<th>Allowable Item Types*</th>
<th>SR, CR, ER, TE</th>
</tr>
</thead>
</table>

**Task Models:**

**Problems in pure mathematics.** These are well-posed problems within mathematics where the student must find an approach, choose which mathematical tools to use, carry the solution through, and explain the results.

**Design problems.** These are well-posed problems within a real- or fantasy-world context where the student must find an approach, choose which mathematical tools to use, carry the solution through, and explain the results.

**Planning problems.** Planning problems involve the coordinated analysis of time, space, cost, and people. They are design tasks with a time dimension added. Well-posed problems of this kind assess the student's ability to make the connections needed between different parts of mathematics.

Note: This is not a complete list; other types of tasks that fit the criteria above may be included.

[ Writers/developers should look for other kinds of evidence that support the Claim 2 targets. ]
<table>
<thead>
<tr>
<th>Allowable Tools:</th>
<th>protractor, ruler, calculator, spreadsheet, mathematical software</th>
</tr>
</thead>
<tbody>
<tr>
<td>Key Nontargeted Constructs:</td>
<td>[Note: This cell identifies knowledge and skills the student needs in order to respond, but which are not scored for specified target(s).]</td>
</tr>
<tr>
<td>Claim-Specific Attributes:</td>
<td>Items/tasks must be real-world or scenario-based (e.g., fantasy) and should take 5-15 minutes to solve.</td>
</tr>
<tr>
<td>Accessibility Concerns:</td>
<td>Real-world problems may sometimes be text-heavy. Translation tools and dictionaries should be available to ELL students. Text readers should be available to students, as necessary.</td>
</tr>
<tr>
<td>Sample items:</td>
<td>[Note: Item codes will be hot-button links to sample items that illustrate Claim 2.]</td>
</tr>
</tbody>
</table>

*SR = selected-response item; CR = constructed-response item; TE = technology-enhanced item; ER = extended-response item; PT = performance task

**How to Complete the Item Form for Claim 2**

Items/tasks that are written to Claim 2 assessment targets must follow the guidelines contained throughout these specifications. Additionally, item writers must complete an Item Form for every submitted item/task. Figure 7 provides the model used for all Claim 2 items/tasks, along with an explanation of the metadata that populates each cell of the form.
Figure 7.

<table>
<thead>
<tr>
<th>Sample Item ID:</th>
<th>MAT.GR.IT.2.CDOMA.T.xxx (see below)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Grade:</td>
<td>Specify the 2-digit grade level (HS for high school).</td>
</tr>
<tr>
<td>Primary Claim:</td>
<td><strong>Claim 2: Problem Solving</strong> Students can solve a range of well-posed problems in pure and applied mathematics, making productive use of knowledge and problem-solving strategies.</td>
</tr>
<tr>
<td>Secondary Claim(s):</td>
<td>List Claim 1 targets first, then Claim 3 or 4 targets (as needed), in order of prominence. May be left blank.</td>
</tr>
<tr>
<td>Primary Content Domain:</td>
<td>List the primary content domain of the item/task, as specified in the CCSSM.</td>
</tr>
<tr>
<td>Secondary Content Domain(s):</td>
<td>List additional content domains of the given item/task, as needed. May be left blank.</td>
</tr>
<tr>
<td>Assessment Target(s):</td>
<td>Multiple targets should be listed in order of prominence as related to the item/task. List the claim number first, then the target associated with that claim, accompanied by the text of the target (e.g., 2 C: Interpret results in the context of a situation).</td>
</tr>
<tr>
<td>Standard(s):</td>
<td>Enter the number(s) of the CCSSM standard(s).</td>
</tr>
<tr>
<td>Mathematical Practice(s):</td>
<td>Specify the mathematical practices (1-8) associated with the item/task.</td>
</tr>
<tr>
<td>DOK:</td>
<td>Specify the Depth of Knowledge level (1-4) of the item/task.</td>
</tr>
<tr>
<td>Item Type:</td>
<td>Specify the item type (SR, CR, ER, TE).</td>
</tr>
<tr>
<td>Score Points:</td>
<td>Specify the total point value of the item.</td>
</tr>
<tr>
<td>Difficulty:</td>
<td>Specify the estimated difficulty of item (L=Low, M=Medium, H=Hard). See below for further explanation of coding.</td>
</tr>
<tr>
<td>Key:</td>
<td>Specify the correct key for SR items or indicate “See Sample Top-Score Response” for multi-point items/tasks.</td>
</tr>
<tr>
<td>Stimulus/Source:</td>
<td>Specify any stimulus material used and/or source required for factual information. All sources must be reliable and reproducible. If none, leave blank.</td>
</tr>
<tr>
<td>Target-Specific Attributes (e.g., accessibility issues):</td>
<td>Specify any target-specific attributes (e.g., accessibility issues).</td>
</tr>
<tr>
<td>Notes:</td>
<td>Add any notes here that you believe will aid in understanding the purpose of this sample item. For TE items, include the TE template name here.</td>
</tr>
</tbody>
</table>

**Sample Item ID:** Specify the sample item ID “MAT.GR.IT.C.CDOMA.T.xxx”
- MAT – Mathematics
- GR – 03, 04, 05, 06, 07, 08, or HS
- IT – Item type (SR, CR, TE, ER, or PT)
- C – Claim number 1, 2, 3, or 4
- CDOMA – Content Domain letters from CCSS (e.g., OA, MD, must be five places, lead with zeros until all five places are filled)

**Note:** For PTs, the content domain field is filled with the task name abbreviation.
- T – Assessment target alpha (A, B, C, D, etc.)
- xxx – Item number. Leave alone for now; will be assigned after acceptance.

**Difficulty** – Base the level on the percent of students that would be expected to get the item/task correct or would earn the maximum number of points as follows:
- Low – greater than 70%
- Medium – 40% to 70%
- Hard – less than 40%

*These are all intended to be “hot-button” links to the complete text of each component.*
**Item/Task:**

Use this space for the stem, stimulus, and/or options. The font size for items/tasks is Verdana, 14-point.

---

**Key and Distractor Analysis or Scoring Rubric for Multi-Part Items/Tasks:**

The information in this section will vary, dependent on the item type.

For traditional 1-point SR items: Include the key in both the item form and in this section, and provide complete rationales for each distractor.

For multi-point SR items: Include a rubric and justification for each score point. Refer to the details presented in Figure 3.

For CR and ER items/tasks: Provide a sample top-score response, followed by a rubric and justification for each score point.

For TE items/tasks: Provide a sample top-score response and description of the interaction(s) the student must demonstrate to score each point (may be only 1 point).

**Sample Top-Score Response:**

Provide an example of a complete and thorough top-score response. The language of this response does not need to be “kid-speak,” but it should model what is expected from a student at the specified grade.

---

**Scoring Rubric:**

The language of the rubric should:

- focus on the essence of the primary claim;
- address the requirements of the specific target(s);
- distinguish between different levels of understanding and/or performance; and
- contain relevant information/details/numbers that support different levels of competency related to the item/task.
Claim 3: Communicating Reason

Claim 3 — Students can clearly and precisely construct viable arguments to support their own reasoning and to critique the reasoning of others.

Claim 3 Overview

Claim 3 refers to a recurring theme in the CCSSM content and practice standards—the ability to construct and present a clear, logical, convincing argument. For older students, this may take the form of a rigorous, deductive proof based on clearly stated axioms. For younger students, this will involve more informal justifications. Assessment tasks that address this claim will typically present a claim and ask students to provide, for example, a justification or counterexample.

Essential Properties of Tasks that Assess Claim 3

Rigor is about precision in argument: first avoiding making false statements, then saying more precisely what one assumes, and providing the sequence of deductions one makes on this basis. Assessments should also include tasks that examine a student’s ability to analyze a provided explanation, identify flaws, and correct them. 10

Claim 3 will be assessed using a combination of SR, CR, TE, PT, and ER items/tasks that focus on mathematical reasoning. Some tasks will require students to construct chains of reasoning without specific guidance being provided throughout the task.

Claim 3 items/tasks must draw clearly on knowledge and skills that are articulated in the content standards. At each grade level, the content standards offer natural and productive settings for generating evidence for Claim 3. Items/tasks generating evidence for Claim 3 in a given grade may also draw upon knowledge and skills articulated in the progression of standards up to that grade.

The intent is that each of the targets should not lead to a separate item/task, but provide evidence for several of the assessment targets defined for Claim 3. For this reason, a separate table is not relevant at the target level, so all targets are included in a single grade-level table for Claim 3. For this claim (as with Claims 2 and 4), the statements found in the table cell labeled “Rationale” are drawn from the mathematical practices contained in the CCSSM.

Since Claim 1 specification tables are the only ones in which a direct connection to the content

Figure 8 provides the model used for all Claim 3 tables. Most of the information contained in Figure 8 will be repeated in all Claim 3 tables for high school. Notes have been added to specific cells in order to clarify the information and/or source of the metadata contained in those cells.

**Figure 8.**

<table>
<thead>
<tr>
<th><strong>Primary Claim 3: Communicating Reasoning</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>Students can clearly and precisely construct viable arguments to support their own reasoning and to critique the reasoning of others.</td>
</tr>
<tr>
<td><strong>Secondary Claim(s):</strong> Tasks written primarily to assess Claim 3 will necessarily involve Claim 1 content targets. Related Claim 1 targets should be listed below the Claim 3 targets in the item form. If Claim 2 or Claim 4 targets are also directly related to the task, list those following the Claim 1 targets in order of prominence.</td>
</tr>
<tr>
<td><strong>Primary Content Domain:</strong> Each task should be classified as having a primary, or dominant, content focus. The content should draw upon the knowledge and skills articulated in the progression of standards leading up to high school.</td>
</tr>
<tr>
<td><strong>Secondary Content Domain(s):</strong> While tasks developed to assess Claim 3 will have a primary content focus, components of these tasks will likely produce enough evidence for other content domains that a separate listing of these content domains needs to be included where appropriate.</td>
</tr>
<tr>
<td><strong>Assessment Targets:</strong> Any given task should provide evidence for several of the following assessment targets; each of the following targets should not lead to a separate task. Multiple targets should be listed in order of prominence as related to the task.</td>
</tr>
</tbody>
</table>

**Target A: Test propositions or conjectures with specific examples. (DOK 2)**
Tasks used to assess this target should ask for specific examples to support or refute a proposition or conjecture (e.g., An item stem might begin, "Provide 3 examples to show why/how...").

**Target B: Construct, autonomously,\(^{11}\) chains of reasoning that will justify or refute propositions or conjectures.\(^{12}\) (DOK 3, 4)**
Tasks used to assess this target should ask students to develop a chain of reasoning to justify or refute a conjecture. Tasks for Target B might include the types of examples called for in Target A as part of this reasoning, but they should do so with a lesser degree of scaffolding than tasks that assess Target A alone.

Some tasks for this target will ask students to formulate and justify a conjecture.

---

\(^{11}\) By “autonomous” we mean that the student responds to a single prompt, without further guidance within the task.

\(^{12}\) At the secondary level, these chains may take a successful student 10 minutes to construct and explain. Times will be somewhat shorter for younger students, but still giving them time to think and explain. For a minority of these tasks, subtasks may be constructed to facilitate entry and assess student progress towards expertise. Even for such “apprentice tasks” part of the task will involve a chain of autonomous reasoning that takes at least 5 minutes.
**Target C:** State logical assumptions being used. (DOK 2, 3)
Tasks used to assess this target should ask students to use stated assumptions, definitions, and previously established results in developing their reasoning. In some cases, the task may require students to provide missing information by researching or providing a reasoned estimate.

**Target D:** Use the technique of breaking an argument into cases. (DOK 2, 3)
Tasks used to assess this target should ask students to determine under what conditions an argument is true, to determine under what conditions an argument is not true, or both.

**Target E:** Distinguish correct logic or reasoning from that which is flawed and—if there is a flaw in the argument—explain what it is. (DOK 2, 3, 4)
Tasks used to assess this target present students with one or more flawed arguments and ask students to choose which (if any) is correct, explain the flaws in reasoning, and/or correct flawed reasoning.

**Target F:** Base arguments on concrete referents such as objects, drawings, diagrams, and actions. (DOK 2, 3)
In earlier grades, the desired student response might be in the form of concrete referents. In later grades, concrete referents will often support generalizations as part of the justification rather than constituting the entire expected response.

**Target G:** At later grades, determine conditions under which an argument does and does not apply. (For example, area increases with perimeter for squares, but not for all plane figures.) (DOK 3, 4)
Tasks used to assess this target will ask students to determine whether a proposition or conjecture always applies, sometimes applies, or never applies and provide justification to support their conclusions. Targets A and B will likely be included also in tasks that collect evidence for Target G.

### Relevant Verbs:
Understand, explain, justify, prove, derive, assess, illustrate, and analyze.

(Notes:
* This list of verbs came directly from the Content Specifications, immediately preceding the list of targets for Claim 3, "Relevant Verbs for Identifying Content Clusters and/or Standards for Claim 3.
* These are hot-button links to the full text.
)

### DOK Target(s):
2, 3, 4

**Claim 3 Rationale:**
Mathematical Practice 3: Construct viable arguments and critique the reasoning of others.
Mathematically proficient students:
- understand and use stated assumptions, definitions, and previously established results in constructing arguments.
- make conjectures and build a logical progression of statements to explore the truth of their conjectures.
- analyze situations by breaking them into cases.
- recognize and use counterexamples.
- justify their conclusions, communicate them to others, and respond to the arguments of others.
- reason inductively about data, making plausible arguments that take into account the context from which the data arose.
- compare the effectiveness of plausible arguments.
- distinguish correct logic or reasoning from that which is flawed and, if there is a flaw, explain what it is.
  - Elementary students construct arguments using concrete referents such as objects, drawings, diagrams, and actions.
  - Later students learn to determine domains to which an argument applies.
- listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve arguments.

**Mathematical Practice 6: Attend to precision.**
Mathematically proficient students:
- communicate precisely to others.
- use clear definitions in discussion with others and in their own reasoning.
- state the meaning of the symbols they choose, including using the equal sign consistently and appropriately.
- specify units of measure and label axes to clarify the correspondence with quantities in a problem.
- calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context.
  - In the elementary grades, students give carefully formulated explanations to each other.
  - In high school, students have learned to examine claims and make explicit use of definitions.

<table>
<thead>
<tr>
<th>Allowable Item Types*</th>
<th>SR, CR, ER, TE, PT [Note: These are hot-button links to the full text.]</th>
</tr>
</thead>
</table>
| Task Models:          | **Proof and justification tasks**: These begin with a proposition, and the task is to provide a reasoned argument why the proposition is or is not true. In some tasks, students may be asked to characterize the domain for which the proposition is true.  
  **Critiquing tasks**: Some flawed reasoning is presented, and the task is to correct and improve it.  
  **Mathematical investigations**: Students are presented with a phenomenon and are invited to formulate conjectures about it. They are then asked to prove one of their conjectures.  
  Note: This is not a complete list; other types of tasks that fit the criteria above may be included.  
  [Writers/developers should look for other kinds of evidence that support the Claim 3 targets.] |
| Allowable Tools:      | protractor, ruler, calculator, spreadsheet, mathematical software |
| Key Nontargeted Constructs: | [Note: This cell identifies knowledge and skills the student needs in order to respond, but which are not scored for specified target(s).] |
| Claim-Specific Tasks | Tasks should be designed to take 10-20 minutes to solve. The |
Attributes: computational demand on these tasks should focus on the skill level typically expected for Claim 1 tasks for grades up to the specified grade, yet be consistent with the content domain emphases of that grade.

Accessibility Concerns: Problems that require students to communicate reasoning may sometimes be text-heavy. Translation tools and dictionaries should be available to ELL students. Text readers should be available to students as necessary.

Sample Items: [Note: Item codes will be hot-button links to sample items that illustrate Claim 3.]

*SR = selected-response item; CR = constructed-response item; TE = technology-enhanced item; ER = extended-response item; PT = performance task

**How to Complete the Item Form for Claim 3**

Items/tasks that are written to Claim 3 assessment targets must follow the guidelines contained throughout these specifications. Additionally, item writers must complete an Item Form for every submitted item/task. Figure 9 provides the model used for all Claim 3 items/tasks along with an explanation of the metadata that populates each cell of the form.
### Sample Item ID:

**MAT.GR.IT.3.CDOMA.T.xxx**

**Grade:** Specify the 2-digit grade level (HS for high school).

**Primary Claim:**

**Claim 3: Communicating Reasoning**

Students can clearly and precisely construct viable arguments to support their own reasoning and to critique the reasoning of others.

**Secondary Claim(s):**

List Claim 1 targets first, then Claim 2 or 4 targets (as needed), in order of prominence. May be left blank.

**Primary Content Domain:**

List the primary content domain of the task, as specified in the CCSSM.

**Secondary Content Domain(s):**

List additional content domains of the given task, as needed. May be left blank.

**Assessment Target(s):**

Multiple targets should be listed in order of prominence as related to the item/task. List the claim number first, then the target associated with that claim, accompanied by the text of the target (e.g., 3 C: State logical assumptions being used.)

**Standard(s):**

Enter the number(s) of the CCSSM standard(s).

**Mathematical Practice(s):**

Specify the mathematical practices (1-8) associated with the item/task.

**DOK:**

Specify the Depth of Knowledge level (1-4) of the item/task.

**Item Type:**

Specify the item type (SR, CR, ER, TE, PT).

**Score Points:**

Specify the total point value of the item.

**Difficulty:**

Specify the estimated difficulty of item (L=Low, M=Medium, H=Hard). See below for further explanation of coding.

**Key:**

Specify “See Sample Top-Score Response” for Claim 3 tasks.

**Stimulus/Source:**

Specify any stimulus material used and/or source required for factual information. All sources must be reliable and reproducible. If none, leave blank.

**Claim-Specific Attributes (e.g., accessibility issues):**

Specify any target-specific attributes (e.g., accessibility issues).

**Notes:**

Add any notes here that you believe will aid in understanding the purpose of this sample item. For TE items, include the TE template name here.

---

**Sample item ID:** Specify the sample item ID "MAT.GR.IT.C.CDOMA.T.xxx"

**MAT – Mathematics**

GR – 03, 04, 05, 06, 07, 08, or HS

IT – Item type (SR, CR, TE, ER, or PT)

C – Claim number 1, 2, 3, or 4

CDOMA – Content Domain letters from CCSS (e.g., OA, MD, must be five places, lead with zeros until all five places are filled)

**Note:** For PTs, the content domain field is filled with the task name abbreviation.

**T – Assessment target alpha (A, B, C, D, etc.)

xxx – Item number. Leave alone for now; will be assigned after acceptance.

**Difficulty** – Base the level on the percent of students that would be expected to get the item/task correct or to earn the maximum number of points as follows: Low – greater than 70%; Medium – 40% to 70%; Hard – less than 40%
**Item/Task:**

Use this space for the stem, stimulus, and/or options. The font size for items/tasks is Verdana, 14-point.

---

**Sample Top-Score Response:**

Provide an example of a complete and thorough top-score response. The language of this response does not need to be “kid-speak,” but it should model what is expected from a student at the specified grade.

---

**Scoring Rubric:**

The language of the rubric should:

- focus on the essence of the primary claim;
- address the requirements of the specific target(s);
- distinguish between different levels of understanding and/or performance; and
- contain relevant information/details/numbers that support different levels of competency related to the item/task.

Additionally, the scoring rubric should reflect the values set out for Claim 3, giving substantial weight to the quality and precision of the reasoning in several of the following:

- an explanation of the assumptions made;
- the construction of conjectures that appear plausible, where appropriate;
- the quality of the examples that the student constructs in order to evaluate the proposition or conjecture;
- the reasoning that the student uses to describe flaws or gaps in an argument;
- the clarity and precision with which the student constructs a logical sequence of steps to show how the assumptions lead to the acceptance or refutation of a proposition or conjecture;
- the precision with which the student describes the domain of validity of the proposition or conjecture.
Claim 4: Modeling and Data Analysis

Claim 4 — Students can analyze complex, real-world scenarios and can construct and use mathematical models to interpret and solve problems.

Claim 4 Overview

Modeling is the bridge across the “school math”/“real world” divide that has been missing from many mathematics curricula and assessments. It is the twin of mathematical literacy, the focus of the PISA international comparison tests in mathematics. CCSSM features modeling as both a mathematical practice at all grades and a content focus in high school.

Essential Properties of Tasks that Assess Claim 4

In the real world, problems do not come neatly “packaged.” Real-world problems are complex and often contain insufficient or superfluous data. Tasks designed primarily to assess Claim 4 will involve formulating a problem that is tractable using mathematics; that is, formulating a model. This will usually involve making assumptions and simplifications. Students will need to select from the data at hand or estimate data that are missing. (Such tasks are therefore distinct from the well-formulated problem-solving tasks described in Claim 2.) Students will identify variables in a situation and construct relationships between them. Once students have formulated the problem, they will tackle it (often in a decontextualized manner) before interpreting their results and then checking the results for reasonableness.

Claim 4 tasks will often involve more than one content domain and will draw upon knowledge and skills articulated in the progression of standards up to that grade, with strong emphasis on the major work of previous grades.

Claim 4 will be assessed both by performance tasks (each lasting up to 120 minutes) and by a collection of 6 to 9 extended-response items/tasks which focus on modeling and data analysis. ER tasks should be designed so that a successful student will complete them in 10-20 minutes.

The intent is that each of the Claim 4 targets should not lead to a separate task, but provide evidence for several of the assessment targets defined for Claim 4. For this reason, a separate table is not relevant at the target level, so all targets are included in a single grade-level table for Claim 4. For this claim (as with Claims 2 and 3), the statements found in the table cell labeled “Rationale” are drawn from the mathematical practices contained in the CCSSM.

Since Claim 1 specification tables are the only ones in which a direct connection to the content domains and clusters of the grade-level CCSSM is made, tasks designed to elicit the evidence sought in Claim 4 will necessarily rely on the content explicated in the Claim 1 specification tables.

Figure 10 provides the model used for all Claim 4 tables. Most of the information contained in Figure 10 will be repeated in all Claim 4 tables for high school. Notes have been added to specific cells in order to clarify the information and/or source of the metadata contained in those cells.

13 In their everyday life and work, most adults use none of the mathematics they are first taught after age 11. They often do not see the mathematics that they do use (in planning, personal accounting, design, thinking about political issues, etc.) as mathematics.
Primary Claim 4: Modeling and Data Analysis
Students can analyze complex, real-world scenarios and can construct and use mathematical models to interpret and solve problems.

Secondary Claim(s): Tasks written primarily to assess Claim 4 will necessarily involve Claim 1 content targets. Related Claim 1 targets should be listed below the Claim 4 targets in the item form. If Claim 2 or Claim 3 targets are also directly related to the task, list those following the Claim 1 targets in order of prominence.

Primary Content Domain: Each task should be classified as having a primary, or dominant, content focus. The content should draw upon the knowledge and skills articulated in the progression of standards leading up to high school, with strong emphasis on the major work of previous grades.

Secondary Content Domain(s): While tasks developed to assess Claim 4 will have a primary content focus, components of these tasks will likely produce enough evidence for other content domains that a separate listing of these content domains needs to be included where appropriate.

Assessment Targets: Any given task should provide evidence for several of the following assessment targets; each of the following targets should not lead to a separate task. Multiple targets should be listed in order of prominence as related to the task.

Target A: Apply mathematics to solve problems arising in everyday life, society, and the workplace. (DOK 2, 3)
Problems used to assess this target for Claim 4 should not be completely formulated (as they are for the same target in Claim 2), and require students to extract relevant information from within the problem and find missing information through research or the use of reasoned estimates.

Target B: Construct, autonomously, chains of reasoning to justify mathematical models used, interpretations made, and solutions proposed for a complex problem. (DOK 2, 3, 4)
At the secondary level, these chains should typically take a successful student 10 minutes to complete. Times will be somewhat shorter for younger students, but still give them time to think and explain. For a minority of these tasks, subtasks may be constructed to facilitate entry and assess student progress towards expertise. Even for such “apprentice tasks,” part of the task will involve a chain of autonomous reasoning that takes at least 5 minutes.

Target C: State logical assumptions being used. (DOK 1, 2)
Tasks used to assess this target ask students to use stated assumptions, definitions, and previously established results in developing their reasoning. In some cases, the task may require students to provide missing information by researching or providing a reasoned estimate.

Target D: Interpret results in the context of a situation. (DOK 2, 3)
Tasks used to assess this target should ask students to link their answer(s) back to the problem’s context. (See Claim 2, Target C, for further explication.)

Target E: Analyze the adequacy of and make improvements to an existing model or develop a mathematical model of a real phenomenon. (DOK 3, 4)
Tasks used to assess this target ask students to investigate the efficacy of existing models.
(e.g., develop a way to analyze the claim that a child’s height at age 2 doubled equals his/her adult height) and suggest improvements using their own or provided data.

Other tasks for this target will ask students to develop a model for a particular phenomenon (e.g., analyze the rate of global ice melt over the past several decades and predict what this rate might be in the future).

Longer constructed-response items and extended performance tasks should be used to assess this target.

**Target F: Identify important quantities in a practical situation and map their relationships (e.g., using diagrams, two-way tables, graphs, flowcharts, or formulas). (DOK 1, 2, 3)**

Unlike Claim 2, where this target might appear as a separate target of assessment (see Claim 2, Target D), it will be embedded in a larger context for tasks in Claim 4. The mapping of relationships should be part of the problem posing and solving related to Claim 4, Targets A, B, E, and G.

**Target G: Identify, analyze and synthesize relevant external resources to pose or solve problems. (DOK 3, 4)**

Especially in extended performance tasks (those requiring up to 120 minutes to complete), students should have access to external resources to support their work in posing and solving problems (e.g., finding or constructing a set of data or information to answer a particular question or looking up measurements of a structure to increase precision in an estimate for a scale drawing). Constructed-response items should incorporate “hyperlinked” information to provide additional detail (both relevant and extraneous) for solving problems in Claim 4.

**Relevant Verbs:**
- model, construct, compare, investigate, build, interpret, estimate, analyze, summarize, represent, solve, evaluate, extend, and apply.

*[Note: This list of verbs came directly from the Content Specifications, immediately preceding the list of targets for Claim 4, “Relevant Verbs for Identifying Content Clusters and/or Standards for Claim 4.”]*

**DOK Target(s):**
- 1, 2, 3, 4  
  *[Note: These are hot-button links to the full text.]*

**Claim 4 Rationale:**

**Mathematical Practice 2: Reason abstractly and quantitatively.**

Mathematically proficient students:
- make sense of quantities and their relationships in problem situations.
- bring two complementary abilities to bear on problems involving quantitative relationships:
  - Decontextualize (abstract a given situation and represent it symbolically; and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents) and
  - Contextualize (pause as needed during the manipulation process in order to probe into the referents for the symbols involved).
- use quantitative reasoning that entails creating a coherent representation of the problem at hand, considering the units involved, and attending to the meaning of quantities (not just how to compute them).
• know and flexibly use different properties of operations and objects.

Mathematical Practice 4: Model with mathematics.
Mathematically proficient students:
• apply the mathematics they know to solve problems arising in everyday life, society, and the workplace.
  o In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community.
  o By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another.
• make assumptions and approximations to simplify a complicated situation, realizing that these may need revision later.
• identify important quantities in a practical situation.
• map relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas.
• analyze those relationships mathematically to draw conclusions.
• interpret their mathematical results in the context of the situation.
• reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

Mathematical Practice 5: Use appropriate tools strategically.
Mathematically proficient students:
• consider available tools when solving a mathematical problem. (Tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software.)
• are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations.
• detect possible errors by using estimations and other mathematical knowledge.
• know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data.
• identify relevant mathematical resources and use them to pose or solve problems.
• use technological tools to explore and deepen their understanding of concepts.

<table>
<thead>
<tr>
<th>Allowable Item Types*</th>
<th>PT, CR, ER, TE [Note: These are hot-button links to the full text.]</th>
</tr>
</thead>
</table>

Task Models: **Make decisions from data:** These tasks require students to select from a data source, analyze the data, and draw reasonable conclusions from it. This will often result in an *evaluation* or *recommendation*. The purpose of these tasks is not to provide a setting for the student to demonstrate a particular data analysis skill.
(e.g., box-and-whisker plots); rather, the purpose is the drawing of conclusions in a realistic setting using a range of techniques.

**Make reasoned estimates:** These tasks require students to make reasonable estimates of things they do know, so that they can then build a chain of reasoning that gives them an estimate of something they do not know.

**Plan and design:** Students recognize that this is a problem situation that arises in life and work. Well-posed planning tasks involving the coordinated analysis of time, space, and cost have already been recommended for assessing Claim 2. For Claim 4, the problem will be presented in a more open form, asking the students to identify the variables that need to be taken into account and the information they will need to find.

**Evaluate and recommend:** These tasks involve understanding a model of a situation and/or some data about it and making a recommendation.

**Interpret and critique:** These tasks involve interpreting some data and critiquing an argument based on it. Again, the purpose of these tasks is not to provide a setting for the student to demonstrate a particular data analysis skill, but to draw conclusions in a realistic setting using a range of techniques.

Note: This is not a complete list; other types of tasks that fit the criteria above may be included.

> [Writers/developers should look for other kinds of evidence that support the Claim 4 targets.]

<table>
<thead>
<tr>
<th>Allowable Tools:</th>
<th>protractor, ruler, calculator, spreadsheet, mathematical software</th>
</tr>
</thead>
<tbody>
<tr>
<td>Key Nontargeted Constructs:</td>
<td>[Note: This cell identifies knowledge and skills the student needs in order to respond, but which are not scored for specified target(s).]</td>
</tr>
<tr>
<td>Claim-Specific Attributes:</td>
<td>CR, ER, and TE tasks should be designed to take 10-20 minutes to solve, while PTs may take up to 120 minutes.</td>
</tr>
<tr>
<td>Accessibility Concerns:</td>
<td>Problems that require students to communicate reasoning may sometimes be text-heavy. Translation tools and dictionaries should be available to ELL students. Text readers should be available to students</td>
</tr>
</tbody>
</table>
Sample Items:  
[Note: Item codes will be hot-button links to sample items that illustrate Claim 4.]

<table>
<thead>
<tr>
<th>Sample Items:</th>
</tr>
</thead>
<tbody>
<tr>
<td>as necessary.</td>
</tr>
</tbody>
</table>

*SR = selected-response item; CR = constructed-response item; TE = technology-enhanced item; ER = extended-response item; PT = performance task

**How to Complete the Item Form for Claim 4**

Tasks that are written to Claim 4 assessment targets must follow the guidelines contained throughout these specifications. Additionally, item writers must complete an Item Form for every submitted task. Figure 11 provides the model used for all Claim 4 tasks, along with an explanation of the metadata that populates each cell of the form.
<table>
<thead>
<tr>
<th>Sample Item ID:</th>
<th>MAT.GR.IT.4.CDOMA.T.xxx (see below)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Grade:</td>
<td>Specify the 2-digit grade level (HS for high school).</td>
</tr>
<tr>
<td>Primary Claim:</td>
<td><strong>Claim 4: Modeling and Data Analysis</strong></td>
</tr>
<tr>
<td></td>
<td>Students can analyze complex, real-world scenarios and can construct and use mathematical models to interpret and solve problems.</td>
</tr>
<tr>
<td>Secondary Claim(s):</td>
<td>List Claim 1 targets first, then Claim 2 or 3 targets (as needed), in order of prominence. May be left blank.</td>
</tr>
<tr>
<td>Primary Content Domain:</td>
<td>List the primary content domain of the task, as specified in the CCSSM.</td>
</tr>
<tr>
<td>Secondary Content Domain(s):</td>
<td>List additional content domains of the given task, as needed. May be left blank.</td>
</tr>
<tr>
<td>Assessment Target(s):</td>
<td>Multiple targets should be listed in order of prominence as related to the task. List the claim number first, then the target associated with that claim, accompanied by the text of the target (e.g., 4 D: Interpret results in the context of a situation).</td>
</tr>
<tr>
<td>Standard(s):</td>
<td>Enter the number(s) of the CCSSM standard(s).</td>
</tr>
<tr>
<td>Mathematical Practice(s):</td>
<td>Specify the mathematical practices (1-8) associated with the item/task.</td>
</tr>
<tr>
<td>DOK:</td>
<td>Specify the Depth of Knowledge level (1-4) of the item/task.</td>
</tr>
<tr>
<td>Item Type:</td>
<td>Specify the item type (CR, TE, ER, PT).</td>
</tr>
<tr>
<td>Score Points:</td>
<td>Specify the total point value of the item.</td>
</tr>
<tr>
<td>Difficulty:</td>
<td>Specify the estimated difficulty of item (L=Low, M=Medium, H=Hard). See below for further explanation of coding.</td>
</tr>
<tr>
<td>Key:</td>
<td>Specify &quot;Sample Top-Score Response&quot; for Claim 4 tasks.</td>
</tr>
<tr>
<td>Stimulus/Source:</td>
<td>Specify any stimulus material used and/or source required for factual information. All sources must be reliable and reproducible. If none, leave blank.</td>
</tr>
<tr>
<td>Claim-specific Attributes (e.g., accessibility issues):</td>
<td>Specify any target-specific attributes (e.g., accessibility issues).</td>
</tr>
<tr>
<td>Notes:</td>
<td>Add any notes here that you believe will aid in understanding the purpose of this sample item. For TE items, include the TE template name here.</td>
</tr>
</tbody>
</table>

**Sample Item ID:** Specify the sample item ID “MAT.GR.IT.C.CDOMA.T.xxx”

**MAT** – Mathematics
**GR** – 03, 04, 05, 06, 07, 08, or HS
**IT** – Item type (SR, CR, TE, ER, or PT)
**C** – Claim number 1, 2, 3, or 4
**CDOMA** – Content Domain letters from CCSS (e.g., OA, MD, must be five places, lead with zeros until all five places are filled)
**T** – Assessment target alpha (A, B, C, D, etc.)
**xxx** – Item number. Leave alone for now; will be assigned after acceptance.

**Note:** For PTs, the content domain field is filled with the task name abbreviation.

**Difficulty** – Base the level on the percent of students that would be expected to get the item/task correct or to earn the maximum number of points as follows: Low – greater than 70%; Medium – 40% to 70%; Hard – less than 40%
**Item/Task:**

Use this space for the stem, stimulus, and/or options. The font size for items/tasks is Verdana, 14-point.

---

**Sample Top-Score Response:**

Provide an example of a complete and thorough top-score response. The language of this response does not need to be “kid-speak,” but it should model what is expected from a student at the specified grade.

---

**Scoring Rubric:**

The scoring rubric for each task should reflect the values set out for this claim, giving substantial weight to the choice of appropriate methods of attacking the task, to reliable skills in carrying it through, and to explanations of what has been found.

Additionally, the language of the rubric should:

- focus on the essence of the primary claim;
- address the requirements of the specific target(s);
- distinguish between different levels of understanding and/or performance; and
- contain relevant information/details/numbers that support different levels of competency related to the item/task.
Appendix A: Smarter Balanced Mathematics Glossary

Grades 3–High School

Many of the terms and definitions in this glossary have been taken directly from the Common Core State Standards for Mathematics (CCSSM) and are indicated with an asterisk (*). The original CCSSM glossary has been supplemented with definitions to aid in the interpretation of the specification tables and item forms found throughout these Specifications. Italicized words or phrases within a definition are defined separately in this glossary.

The Claim 1 Mathematics Specification Tables contain a cell for “allowable disciplinary vocabulary” for each target, by grade. All vocabulary terms referenced in a previous grade is considered expected knowledge for subsequent grades.

**Absolute value**
A number’s distance from zero (0) on a number line. Distance is expressed as a positive value. Example: \( |7| = 7 \) and \( |-7| = 7 \).

**Acute angle**
An angle that measures less than 90° and greater than 0°.

**Addend**
Any number being added.

**Additive identity**
The number zero (0). When zero (0) is added to another number, it does not change the number’s value (e.g., \( 12 + 0 = 12 \)).

**Additive inverses* **
Two numbers whose sum is 0 are additive inverses of one another. Example: \( \frac{3}{4} \) and \( -\frac{3}{4} \) are additive inverses of one another because \( \frac{3}{4} + (-\frac{3}{4}) = 0 \).

**Algebraic equation (inequality)**
An expression containing variables in which two expressions are connected by an equality (inequality) symbol. See also: equation and inequality.

**Algebraic expression**
An expression containing numbers and variables (e.g., \( 3x \)), and operations that involve numbers and variables (e.g., \( 8x + 5y \) or \( 4a^2 - 7b + 13 \)). Algebraic expressions do not contain equality or inequality symbols.

**Algebraic rule**
A mathematical expression that contains variables and describes a pattern or relationship.

**Altitude**
The perpendicular distance from a vertex in a polygon to its opposite side.

**Angle**
Two rays extending from a common end point called the vertex. Angles are measured in degrees.

**Area**
The measure, in square units, of the inside region of a closed two-dimensional figure (e.g., a rectangle with sides of 3 units by 9 units has an area of 27 square units).
Associative property: The way in which three or more numbers are grouped for addition or multiplication does not change their \textit{sum} or \textit{product}, respectively (e.g., \((3 + 1) + 9 = 3 + (1 + 9)\) or \((5 \times 3) \times 10 = 5 \times (3 \times 10)\)).

Axes (of a graph): The horizontal and vertical \textit{number lines} used in a \textit{coordinate plane} system.

Axis: The singular form of \textit{axes}.

Bar graph: A graph that uses either vertical or horizontal bars to \textit{display data}.

Base (algebraic): The number used as a factor in \textit{exponential form}. Example: \(5^3\) is the \textit{exponential form} of \(5 \times 5 \times 5\). The numeral five (5) is called the base, and the numeral three (3) is called the \textit{exponent}.

Base (geometric): The line or plane of a \textit{geometric figure}, from which an \textit{altitude} can be constructed, upon which a figure is thought to rest.

Bivariate data*: Pairs of linked numerical observations. Example: a list of heights and \textit{weights} for each player on a football team.

Box plot*: A method of visually displaying a distribution of data values by using the \textit{median}, quartiles, and extremes of the data set. A box shows the middle 50% of the data.\(^\text{14}\)

Break: A zigzag on the x- or y-axis in a line or bar graph indicating that the data being displayed do not include all of the values that exist on the \textit{number line} used. Also called a \textit{squiggle}.

Capacity: The amount of space that can be filled in a container. Both capacity and \textit{volume} are used to measure three-dimensional spaces; however, capacity usually refers to fluids, whereas \textit{volume} usually refers to solids.

Central angle: An angle that has its vertex at the center of a circle, with \textit{radii} as its sides.

Circle graph: A \textit{data display} that divides a circle into regions representing a portion of the total set of data. The circle represents the whole set of data.

Circumference: The \textit{perimeter} of a circle.

Closed figure: A two-dimensional figure that divides the \textit{plane} in which the figure lies into two parts—the part inside the figure and the part outside the figure (e.g., circles, squares, rectangles).

\(^{14}\) Adapted from Wisconsin Department of Public Instruction, \url{http://dpi.wi.gov/standards/mathglos.html}, accessed March 2, 2010.
Commutative property
The order in which two numbers are added or multiplied does not change their sum or product, respectively (e.g., \(4 + 3 = 3 + 4\) or \(5 \times 7 = 7 \times 5\)).

Complementary angles
Two angles with measures that sum to be exactly 90°.

Complex fraction*
A fraction \(A/B\) where \(A\) and/or \(B\) are fractions (\(B\) nonzero).

Composite number
A whole number that has more than two factors.

Congruent*
Two plane or solid figures are congruent if one can be obtained from the other by rigid motion (a sequence of rotations, reflections, and translations).

Coordinate grid or plane
A two-dimensional network of horizontal and vertical lines that are parallel and evenly-spaced; especially designed for locating points, displaying data, or drawing maps.

Coordinates
Numbers that correspond to points on a coordinate plane in the form \((x, y)\), or a number that corresponds to a point on a number line.

Counting principle
If a first event has \(n\) outcomes and a second event has \(m\) outcomes, then the first event followed by the second event has \(n \times m\) outcomes.

Customary units
The units of measure developed and used in the United States. Customary units for:
- length are inches, feet, yards, and miles.
- weight are ounces, pounds, and tons.
- volume are cubic inches, cubic feet, and cubic yards.
- capacity are fluid ounces, cups, pints, quarts, and gallons.

Cylinder
A three-dimensional figure with two parallel bases that are congruent circles.

Data displays/graphs
Different ways of displaying data in charts, tables, or graphs, including pictographs, circle graphs, single-, double-, or triple-bar and line graphs, histograms, stem-and-leaf plots, box-and-whisker plots, and scatter plots.

Decimal number
Any number written with a decimal point in the number. A decimal number falls between two whole numbers (e.g., 4.7 falls between 5 and 6). Decimal numbers smaller than 1 are sometimes called decimal fractions (e.g., three-tenths is written 0.3).

Dilation*
A transformation that moves each point along the ray through the point emanating from a fixed center, and multiplies distances from the center by a common scale factor.
<table>
<thead>
<tr>
<th>Term</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Diameter</td>
<td>A <em>line segment</em> from any point on the circle passing through the center to another point on the circle.</td>
</tr>
<tr>
<td>Difference</td>
<td>A number that is the result of subtracting two numbers.</td>
</tr>
<tr>
<td>Direct measure</td>
<td>Obtaining the measure of an object by using measuring devices, either standard devices of the <em>customary</em> or <em>metric systems</em>, or nonstandard devices such as a paper clip or pencil.</td>
</tr>
<tr>
<td>Distributive property</td>
<td>The <em>product</em> of a number and the <em>sum</em> or <em>difference</em> of two numbers is equal to the <em>sum</em> or <em>difference</em> of the two <em>products</em>. Example: ( x(a + b) = ax + bx ).</td>
</tr>
<tr>
<td>Divisible</td>
<td>A number capable of being divided into equal parts without a remainder.</td>
</tr>
<tr>
<td>Divisor</td>
<td>The number by which another number is divided.</td>
</tr>
<tr>
<td>Dot plot*</td>
<td>See: <em>line plot</em></td>
</tr>
<tr>
<td>Empirical probability</td>
<td>The likelihood of an event happening that is based on experience and observation rather than on theory.</td>
</tr>
<tr>
<td>Enlargement</td>
<td>See: <em>dilation</em></td>
</tr>
<tr>
<td>Equilateral triangle</td>
<td>A triangle with three <em>congruent</em> sides.</td>
</tr>
<tr>
<td>Equation</td>
<td>A mathematical sentence in which two <em>expressions</em> are connected by an <em>equality</em> symbol. See also <em>algebraic equation</em> (<em>inequality</em>).</td>
</tr>
<tr>
<td>Equivalent expressions</td>
<td><em>Expressions</em> that have the same value but are presented in a different format using the properties of numbers.</td>
</tr>
<tr>
<td>Equivalent forms of a number</td>
<td>The same number expressed in different forms (e.g., ( \frac{1}{4}, 0.25, 25% )).</td>
</tr>
<tr>
<td>Estimation</td>
<td>The use of rounding and/or other strategies to determine a reasonably accurate approximation, without calculating an exact answer (e.g., clustering, front-end estimating, grouping).</td>
</tr>
<tr>
<td>Evaluate an algebraic expression</td>
<td>Substitute numbers for the <em>variables</em> and follow the <em>algebraic order of operations</em> to find the numerical value of the <em>expression</em>.</td>
</tr>
<tr>
<td>Expanded form*</td>
<td>A multi-digit number is expressed in expanded form when it is written as a sum of single-digit multiples of powers of ten (e.g., ( 643 = 600 + 40 + 3 )).</td>
</tr>
<tr>
<td>Expected value*</td>
<td>For a random variable, the weighted average of its possible values, with <em>weights</em> given by their respective probabilities.</td>
</tr>
</tbody>
</table>
Exponent (exponential form)  The number of times the base occurs as a factor. Example: $5^3$ is the exponential form of $5 \times 5 \times 5$. The numeral five (5) is called the base, and the numeral three (3) is called the exponent.

Expression  A collection of numbers, symbols, and/or operation signs that stands for a number.

Extraneous information  Information that is not necessary to solving the problem.

Extrapolate  To estimate or infer a value or quantity beyond the known range of data.

Face  One of the plane surfaces bounding a three-dimensional figure; a side.

Factor  A number or expression that divides evenly into another number [e.g., 1, 2, 4, 5, 10, and 20 are factors of 20 and $(x + 1)$ is one of the factors of $(x^2 - 1)$].

First quartile*  For a data set with median $M$, the first quartile is the median of the data values less than $M$. Example: for the data set {1, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the first quartile is 6. See also: median, third quartile, and interquartile range.

Fraction*  A number expressible in the form $a/b$ where $a$ is a whole number and $b$ is a positive whole number. (The word fraction in these specifications always refers to a non-negative number.) See also: rational number.

Function  A relation in which each value of $x$ is paired with a unique value of $y$.

Function table  A table of $x$- and $y$-values (ordered pairs) that represents the function, pattern, relationship, or sequence between the two variables.

Grid  See: coordinate grid.

Height  A line segment extending from the vertex or apex of a figure to its base and forming a right angle with the base or the plane that contains the base.

Hypotenuse  The longest side of a right triangle; the side opposite the right angle.

Hypothesis  A proposition or supposition developed to provide a basis for further investigation or research.

Independently combined probability models*  Two probability models are said to be combined independently if the probability of each ordered pair in the combined model equals the product of the original probabilities of the two individual outcomes in the ordered pair.

Indirect measure  The measurement of an object through the known measure of another object.
Inequality  A sentence that states one expression is greater than, greater than or equal to, less than, less than or equal to, or not equal to, another expression (e.g., \(a \neq 2\) or \(x < 4\) or \(3y + 5 \geq 12\)). See also algebraic inequality.

Integer*  A number expressible in the form \(a\) or \(-a\) for some whole number \(a\).

Intercept  The value of a variable when all other variables in the equation equal zero (0). On a graph, the values where a function crosses the axes.

Intersection  The point at which two lines meet.

Interquartile range*  A measure of variation in a set of numerical data, the interquartile range is the distance between the first and third quartiles of the data set. Example: For the data set \(\{1, 3, 6, 7, 10, 12, 14, 15, 22, 120\}\), the interquartile range is \(15 - 6 = 9\). See also: first quartile, third quartile.

Inverse operation  An action that undoes a previously applied action. Example: subtraction is the inverse operation of addition.

Irrational number  A real number that cannot be expressed as a ratio of two numbers (e.g., \(\sqrt{3}\)).

Isosceles triangle  A triangle with two congruent sides and two congruent angles.

Labels (for a graph)  The titles given to a graph, the axes of a graph, or to the scales on the axes of a graph.

Length  A one-dimensional measure that is the measurable property of line segments.

Likelihood  The chance that something is likely to happen. See: probability.

Line  A collection of an infinite number of points in a straight pathway with unlimited length and having no width.

Line plot*  A method of visually displaying a distribution of data values where each data value is shown as a dot or mark above a number line. Also known as a dot plot.\(^{15}\)

Linear equation  An algebraic equation in which the variable quantity or quantities are in the first power only and the graph is a straight line. Examples:  \(40 = 5(x + 1) + 2y; y = 6x + 11\).

\(^{15}\) Adapted from Wisconsin Department of Public Instruction, op. cit.
Linear inequality

An algebraic inequality in which the variable quantity or quantities are in the first power only and the graph is a region whose boundary is the straight line formed by the inequality.

Line graph

A graph that displays data using connected line segments.

Line segment

A portion of a line that consists of a defined beginning and endpoint and all the points in between.

Mass

The amount of matter in an object.

Mean*

A measure of center in a set of numerical data, computed by adding the values in a list and then dividing by the number of values in the list.\textsuperscript{16} Example: For the data set \{1, 3, 6, 7, 10, 12, 14, 15, 22, 120\}, the mean is 21.

Mean absolute deviation*

A measure of variation in a set of numerical data, computed by adding the distances between each data value and the mean, then dividing by the number of data values. Example: for the data set \{2, 3, 6, 7, 10, 12, 14, 15, 22, 120\}, the mean absolute deviation is 20.

Median*

A measure of center in a set of numerical data. The median of a list of values is the value appearing at the center of a sorted version of the list — or the mean of the two central values, if the list contains an even number of values. Example: for the data set \{2, 3, 6, 7, 10, 12, 14, 15, 22, 90\}, the median is 11.

Metric units

The units of measure developed in Europe and used in most of the world. Like the decimal system, the metric system uses the base 10. Metric units for:

- \textit{length} are millimeters, centimeters, meters, and kilometers.
- \textit{mass} are milligrams, grams, and kilograms.
- \textit{volume} are cubic millimeters, cubic centimeters, and cubic meters.
- \textit{capacity} are milliliters, centiliters, liters, and kiloliters.

Midpoint of a line segment

The point on a line segment that divides it into two equal parts.

Midline*

In the graph of a trigonometric function, the horizontal line halfway between its maximum and minimum values.

Multiples

The numbers that result from multiplying a given whole number by the set of whole numbers (e.g., the multiples of 12 are 0, 12, 24, 36, 48, 60).

\textsuperscript{16} To be more precise, this defines the arithmetic mean.
<table>
<thead>
<tr>
<th>Term</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Multiplicative identity</td>
<td>The number one (1). Multiplying by 1 does not change a number’s value (e.g., $8 \times 1 = 8$).</td>
</tr>
<tr>
<td>Multiplicative inverses*</td>
<td>Two numbers whose product is 1 are multiplicative inverses of one another. Example: $\frac{3}{4}$ and $\frac{4}{3}$ are multiplicative inverses of one another because $\frac{3}{4} \times \frac{4}{3} = \frac{4}{3} \times \frac{3}{4} = 1$.</td>
</tr>
<tr>
<td>Natural numbers (counting numbers)</td>
<td>The numbers in the set ${1, 2, 3, 4, 5 \ldots }$.</td>
</tr>
<tr>
<td>Negative exponent</td>
<td>Used to designate the reciprocal of a number to the absolute value of the exponent. Also used in scientific notation to designate a number smaller than one (1). Example: $4.21 \times 10^{-2}$ equals 0.0421.</td>
</tr>
<tr>
<td>Nonstandard units of measure</td>
<td>Objects such as blocks, paper clips, crayons, or pencils that can be used to obtain a measure.</td>
</tr>
<tr>
<td>Number line diagram*</td>
<td>A diagram of the number line used to represent numbers and support reasoning about them. In a number line diagram for measurement quantities, the interval from 0 to 1 on the diagram represents the unit of measure for the quantity.</td>
</tr>
<tr>
<td>Obtuse angle</td>
<td>An angle with a measure of more than $90^\circ$ but less than $180^\circ$.</td>
</tr>
<tr>
<td>Odds</td>
<td>The ratio of one event occurring (favorable outcome) to it not occurring (unfavorable outcome) if all outcomes are equally likely.</td>
</tr>
<tr>
<td>Operation</td>
<td>Any mathematical process, such as addition, subtraction, multiplication, division, raising to a power, or finding the square root.</td>
</tr>
<tr>
<td>Ordered pair</td>
<td>The location of a single point on a rectangular coordinate system, where the digits represent the position relative to the x-axis and y-axis [e.g., (x, y) or (1, -2)].</td>
</tr>
<tr>
<td>Organized data</td>
<td>Data arranged in a display that is meaningful and assists in the interpretation of the data. See: data displays.</td>
</tr>
<tr>
<td>Origin</td>
<td>The point of intersection of the x- and y-axes in a rectangular coordinate system, where the x-coordinate and y-coordinate are both zero (0).</td>
</tr>
<tr>
<td>Parallel lines</td>
<td>Two lines in the same plane that are a constant distance apart. Parallel lines never meet and have equal slopes.</td>
</tr>
<tr>
<td>Pattern (relationship)</td>
<td>A predictable or prescribed sequence of numbers, objects, etc. Patterns and relationships may be described or presented using manipulatives, tables, graphics (pictures or drawings), or algebraic rules (functions).</td>
</tr>
<tr>
<td>Percent</td>
<td>A special-case ratio which compares numbers to 100 (the second term). Example: 75% means the ratio of 75 to 100.</td>
</tr>
<tr>
<td>Term</td>
<td>Definition</td>
</tr>
<tr>
<td>-------------------------------</td>
<td>------------------------------------------------------------------------------------------------------------------------------------------</td>
</tr>
<tr>
<td>Percent rate of change*</td>
<td>A rate of change expressed as a percent. Example: if a population grows from 50 to 55 in a year, it grows by 5/50 = 10% per year.</td>
</tr>
<tr>
<td>Perimeter</td>
<td>The distance around a polygon.</td>
</tr>
<tr>
<td>Perpendicular</td>
<td>Two lines, two line segments, or two planes that cross to form a right angle.</td>
</tr>
<tr>
<td>PI (π)</td>
<td>The symbol designating the ratio of the circumference of a circle to its diameter. It is an irrational number with common approximations of either 3.14 or 22/7.</td>
</tr>
<tr>
<td>Place value</td>
<td>The position of a single digit in a number.</td>
</tr>
<tr>
<td>Planar cross-section</td>
<td>The intersection of a plane and a three-dimensional figure.</td>
</tr>
<tr>
<td>Plane</td>
<td>An undefined, two-dimensional geometric surface that has no depth and no boundaries specified. A plane is determined by defining at least three distinct points or at least two distinct lines existing on the plane.</td>
</tr>
<tr>
<td>Plane figure</td>
<td>A two-dimensional figure that lies entirely within a single plane.</td>
</tr>
<tr>
<td>Point</td>
<td>A specific location in space that has no discernible length or width.</td>
</tr>
<tr>
<td>Polygon</td>
<td>A closed-plane figure, having at least three sides that are line segments and are connected at their end-points.</td>
</tr>
<tr>
<td>Prime number</td>
<td>Any whole number with only two whole number factors, 1 and itself (e.g., 2, 3, 5, 7, 11).</td>
</tr>
<tr>
<td>Probability*</td>
<td>A number between 0 and 1 used to quantify likelihood for processes that have uncertain outcomes (such as tossing a coin, selecting a person at random from a group of people, tossing a ball at a target, or testing for a medical condition).</td>
</tr>
<tr>
<td>Probability distribution*</td>
<td>The set of possible values of a random variable with a probability assigned to each.</td>
</tr>
<tr>
<td>Probability model*</td>
<td>A probability model is used to assign probabilities to outcomes of a chance process by examining the nature of the process. The set of all outcomes is called the sample space, and their probabilities sum to 1. See also: uniform probability model.</td>
</tr>
<tr>
<td>Product</td>
<td>The result of multiplying numbers together.</td>
</tr>
<tr>
<td>Proof</td>
<td>A logical argument that demonstrates the truth of a given statement. In a formal proof, each step can be justified with a reason; such as a given, a definition, an axiom, or a previously proven property or theorem.</td>
</tr>
<tr>
<td>Proportion</td>
<td>A mathematical sentence stating that two ratios are equal.</td>
</tr>
<tr>
<td>Term</td>
<td>Definition</td>
</tr>
<tr>
<td>-------------------------------</td>
<td>----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------</td>
</tr>
<tr>
<td>Proportional</td>
<td>Having the same or a constant <em>ratio</em>. Two quantities that have the same <em>ratio</em> are considered directly proportional. Example: if $y = kx$, then $y$ is said to be directly proportional to $x$ and the constant of proportionality is $k$. Two quantities whose <em>products</em> are always the same are considered inversely proportional. Example: if $xy = k$, then $y$ is said to be inversely proportional to $x$.</td>
</tr>
<tr>
<td>Pyramid</td>
<td>A three-dimensional figure whose base is a <em>polygon</em> and whose faces are triangles with a common vertex.</td>
</tr>
<tr>
<td>Pythagorean theorem</td>
<td>The square of the hypotenuse (c) of a right triangle is equal to the sum of the square of the legs (a and b), as shown in the equation $c^2 = a^2 + b^2$.</td>
</tr>
<tr>
<td>Quadrant</td>
<td>Any of the four regions formed by the axes in a rectangular coordinate system.</td>
</tr>
<tr>
<td>Quotient</td>
<td>The result of dividing two numbers.</td>
</tr>
<tr>
<td>Radical</td>
<td>An expression that has a root (e.g., square root, cube root). Example: $\sqrt{36}$ is a radical. Any root can be specified by an index number, $b$, in the form $\sqrt[b]{a}$ (e.g., $\sqrt[3]{27}$). A radical without an index number is understood to be a square root.</td>
</tr>
<tr>
<td>Radical sign</td>
<td>The symbol ($\sqrt{}$) used before a number to show that the number is a radicand. See also: radical.</td>
</tr>
<tr>
<td>Radicand</td>
<td>The number that appears within a radical sign (e.g., in $\sqrt{36}$, 36 is the radicand).</td>
</tr>
<tr>
<td>Radius</td>
<td>A line segment extending from the center of a circle or sphere to a point on the circle or sphere. Plural: radii.</td>
</tr>
<tr>
<td>Randomly (chosen)</td>
<td>An equal chance of being chosen.</td>
</tr>
<tr>
<td>Range</td>
<td>The lowest value (L) in a set of numbers through the highest value (H) in the set. When the width of the range is expressed as a single number, the range is calculated as the difference between the highest and lowest values (H – L). Other presentations show the range calculated as (H – L + 1). Depending on the context, the result of either calculation would be considered correct.</td>
</tr>
<tr>
<td>Rate</td>
<td>A <em>ratio</em> that compares two quantities of different units (e.g., miles per hour).</td>
</tr>
<tr>
<td>Ratio</td>
<td>The comparison of two quantities (e.g., the ratio of $a$ and $b$ is $a:b$ or $a/b$, where $b \neq 0$).</td>
</tr>
<tr>
<td>Term</td>
<td>Definition</td>
</tr>
<tr>
<td>------------------------------------</td>
<td>---------------------------------------------------------------------------</td>
</tr>
<tr>
<td>Rational expression*</td>
<td>A quotient of two polynomials with a non-zero denominator.</td>
</tr>
<tr>
<td>Rational number*</td>
<td>A number expressible in the form $a/b$ or $-a/b$ for some fraction $a/b$.</td>
</tr>
<tr>
<td></td>
<td>The rational numbers include the integers.</td>
</tr>
<tr>
<td>Ray</td>
<td>A portion of a line that begins at an endpoint and goes on indefinitely in</td>
</tr>
<tr>
<td></td>
<td>one direction.</td>
</tr>
<tr>
<td>Real numbers</td>
<td>The set of all rational and irrational numbers.</td>
</tr>
<tr>
<td>Reciprocal</td>
<td>See: multiplicative inverse.</td>
</tr>
<tr>
<td>Rectangular coordinate system</td>
<td>See: Coordinate grid or plane.</td>
</tr>
<tr>
<td>Rectilinear figure*</td>
<td>A polygon, all angles of which are right angles.</td>
</tr>
<tr>
<td>Reduction</td>
<td>See: dilation.</td>
</tr>
<tr>
<td>Reflection</td>
<td>A transformation that produces the mirror image of a geometric figure over</td>
</tr>
<tr>
<td></td>
<td>a line of reflection.</td>
</tr>
<tr>
<td>Reflexive property of equality</td>
<td>A number or expression is equal to itself (e.g., $3 = 3$ or $ab = ab$).</td>
</tr>
<tr>
<td>Regular polygon</td>
<td>A polygon that is both equilateral (all sides congruent) and equiangular</td>
</tr>
<tr>
<td></td>
<td>(all angles congruent).</td>
</tr>
<tr>
<td>Relation</td>
<td>A set of ordered pairs $(x, y)$.</td>
</tr>
<tr>
<td>Relative size</td>
<td>The size of one number in comparison to the size of another number or</td>
</tr>
<tr>
<td></td>
<td>numbers.</td>
</tr>
<tr>
<td>Repeating decimal*</td>
<td>The decimal form of a rational number. See also: terminating decimal.</td>
</tr>
<tr>
<td>Right angle</td>
<td>An angle whose measure is exactly 90°.</td>
</tr>
<tr>
<td>Right circular cylinder</td>
<td>A cylinder in which the bases are parallel circles, perpendicular to the</td>
</tr>
<tr>
<td></td>
<td>side of the cylinder.</td>
</tr>
<tr>
<td>Right prism or rectangular solid</td>
<td>A three-dimensional figure (polyhedron) with congruent, polygonal bases</td>
</tr>
<tr>
<td></td>
<td>and lateral faces that are all parallelograms.</td>
</tr>
<tr>
<td>Rigid motion*</td>
<td>A transformation of points in space consisting of a sequence of one or</td>
</tr>
<tr>
<td></td>
<td>more translations, reflections, and/or rotations. Rigid motions here are</td>
</tr>
<tr>
<td></td>
<td>assumed to preserve distances and angle measures.</td>
</tr>
<tr>
<td>Rotation</td>
<td>A transformation of a figure by turning it about a center point or axis.</td>
</tr>
<tr>
<td></td>
<td>The amount of rotation is usually expressed in the number of degrees (e.g.,</td>
</tr>
<tr>
<td></td>
<td>a 90° rotation).</td>
</tr>
</tbody>
</table>
Rule
A mathematical expression that describes a pattern or relationship, or a written description of the pattern or relationship.

Scalene triangle
A triangle having no congruent sides.

Sample space*
A probability model for a random process, a list of the individual outcomes that are to be considered.

Scale factor
The constant that is multiplied by the length of each side of a figure that produces an image that is the same shape as the original figure, but a different size.

Scale model
A model or drawing based on a ratio of the dimensions for the model and the actual object it represents.

Scale
The numeric values, set at fixed intervals, assigned to the axes of a graph.

Scatter plot*
A graph in the coordinate plane representing a set of bivariate data. For example, the heights and weights of a group of people could be displayed on a scatter plot.

Scientific notation
A shorthand method of writing very large or very small numbers using exponents in which a number is expressed as the product of a power of 10 and a number that is greater than or equal to one (1) and less than 10 (e.g., \(4.23 \times 10^6 = 4,230,000\)).

Sequence
An ordered list of numbers with either a constant difference (arithmetic) or a constant ratio (geometric).

Side
The edge of a polygon (e.g., a triangle has three sides) or one of the rays that make up an angle.

Similar figures
Figures that are the same shape, have corresponding congruent angles, and have corresponding sides that are proportional in length.

Similarity
A term describing figures that are the same shape but are not necessarily the same size or in the same position.

Slope
The ratio of change in the vertical axis (y-axis) to each unit change in the horizontal axis (x-axis) in the form rise/run or \(\Delta y/\Delta x\). Also, the constant, \(m\), in the linear equation for the slope-intercept form, \(y = mx + b\).

Solid figures
Three-dimensional figures that completely enclose a portion of space (e.g., a rectangular solid, cube, sphere, right circular cylinder, right circular cone, and square pyramid).

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17 Adapted from Wisconsin Department of Public Instruction, op. cit.
| **Sphere** | A three-dimensional figure in which all points on the figure are equidistant from a center point. |
| **Square root** | A positive real number that can be multiplied by itself to produce a given number (e.g., the square root of 49 is 7 or $\sqrt{49} = 7$). |
| **Squiggle** | See: break. |
| **Standard units of measure** | Accepted measuring devices and units of the customary or metric system. |
| **Stem-and-leaf plot** | A graph that organizes data by place value to compare data frequencies. |
| **Straight angle** | An angle that measures exactly 180°. |
| **Sum** | The result of adding numbers together. |
| **Supplementary angles** | Two angles, the sum of which is exactly 180°. |
| **Surface area of a geometric solid** | The sum of the areas of the faces and any curved surfaces of the figure that create the geometric solid. |
| **Symmetry** | A term describing the result of a line drawn through the center of a figure such that the two halves of the figure are reflections of each other across the line. |
| **System of equations** | A group of two or more equations that are related to the same situation and share variables. The solution to a system of equations is an ordered number set that makes all of the equations true. |
| **Tape diagram** | A drawing that looks like a segment of tape, used to illustrate number relationships. Also known as a strip diagram, bar model, fraction strip, or length model. |
| **Terminating decimal** | A decimal is called terminating if its repeating digit is 0. |
| **Theoretical/expected probability** | The likelihood of an event happening based on theory rather than on experience and observation. |
| **Third quartile** | For a data set with median $M$, the third quartile is the median of the data values greater than $M$. Example: for the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the third quartile is 15. See also: median, first quartile, interquartile range. |
| **Transformation** | An operation on a geometric figure by which another image is created. Common transformations include reflections (flips), translations (slides), rotations (turns) and dilations. |
Transitive property
When the first element has a particular relationship to a second element that in turn has the same relationship to a third element, the first has this same relationship to the third element (e.g., if \( a = b \) and \( b = c \), then \( a = c \)).

Transitivity principle for indirect measurement*
If the length of object A is greater than the length of object B, and the length of object B is greater than the length of object C, then the length of object A is greater than the length of object C. This principle applies to measurement of other quantities as well.

Translation
A transformation in which every point in a figure is moved in the same direction and by the same distance.

Transversal
A line that intersects two or more lines at different points.

Tree diagram
A diagram in which all the possible outcomes of a given event are displayed.

Trend line
A line on a graph that shows a trend between data points.

Uniform probability model*
A probability model which assigns equal probability to all outcomes. See also: probability model.

Unorganized data
Data that are presented in a random manner.

Variable
Any symbol, usually a letter, that could represent a number.

Vector*
A quantity with magnitude and direction in the plane or in space defined by an ordered pair or triple of real numbers.

Vertex
The common endpoint from which two rays begin (i.e., the vertex of an angle) or the point where two lines intersect; the point on a triangle or pyramid opposite to and farthest from the base.

Vertical angles
The opposite or non-adjacent angles formed when two lines intersect.

Visual fraction model*
A tape diagram, number line diagram, or area model.

Volume
The amount of space occupied in three dimensions and expressed in cubic units. Both capacity and volume are used to measure empty spaces; however, capacity usually refers to fluids, whereas volume usually refers to solids.

Weight
Measures that represent the force of gravity on an object.

Whole numbers
The numbers in the set \( \{0, 1, 2, 3, 4 \ldots \} \).

x-axis
The horizontal number line on a rectangular coordinate system.
| **x-intercept** | The value of $x$ at the point where a line or graph intersects the x-axis. The value of $y$ is zero (0) at this point. |
| **y-axis** | The vertical number line on a rectangular coordinate system. |
| **y-intercept** | The value of $y$ at the point where a line or graph intersects the y-axis. The value of $x$ is zero (0) at this point. |
Appendix B: Cognitive Rigor Matrix/Depth of Knowledge (DOK)

The Common Core State Standards require high-level cognitive demand, such as asking students to demonstrate deeper conceptual understanding through the application of content knowledge and skills to new situations and sustained tasks. For each Assessment Target in this document, the depth(s) of knowledge (DOK) that the student needs to bring to the item/task has been identified, using the Cognitive Rigor Matrix shown below. This matrix draws from two widely accepted measures to describe cognitive rigor: Bloom’s (revised) Taxonomy of Educational Objectives and Webb’s Depth-of-Knowledge Levels. The Cognitive Rigor Matrix has been developed to integrate these two models as a strategy for analyzing instruction, for influencing teacher lesson planning, and for designing assessment items and tasks. (To download the full article describing the development and uses of the Cognitive Rigor Matrix and other support CRM materials, go to: http://www.nciea.org/publications/cognitiverigorpaper_KH11.pdf)

A “Snapshot” of the Cognitive Rigor Matrix (Hess, Carlock, Jones, & Walkup, 2009)

<table>
<thead>
<tr>
<th>Depth of Thinking (Webb)+ Type of Thinking (Revised Bloom)</th>
<th>DOK Level 1 Recall &amp; Reproduction</th>
<th>DOK Level 2 Basic Skills &amp; Concepts</th>
<th>DOK Level 3 Strategic Thinking &amp; Reasoning</th>
<th>DOK Level 4 Extended Thinking</th>
</tr>
</thead>
<tbody>
<tr>
<td>Remember</td>
<td>Recall conversions, terms, facts</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Understand</td>
<td>Evaluate an expression</td>
<td>Specify, explain relationships</td>
<td>Use concepts to solve non-routine problems</td>
<td>Relate mathematical concepts to other content areas, other domains</td>
</tr>
<tr>
<td></td>
<td>Locate points on a grid or number on number line</td>
<td>Make basic inferences or logical predictions from data/observations</td>
<td>Use supporting evidence to justify conjectures, generalize, or connect ideas</td>
<td>Develop generalizations of the results obtained and the strategies used and apply them to new problem situations</td>
</tr>
<tr>
<td></td>
<td>Solve a one-step problem</td>
<td>Use models/diagrams to explain concepts</td>
<td>Explain reasoning when more than one response is possible</td>
<td>Explain phenomena in terms of concepts</td>
</tr>
<tr>
<td></td>
<td>Represent math relationships in words, pictures, or symbols</td>
<td>Make and explain estimates</td>
<td>Use supporting evidence to justify conjectures, generalize, or connect ideas</td>
<td>Explain phenomena in terms of concepts</td>
</tr>
<tr>
<td>Apply</td>
<td>Follow simple procedures</td>
<td>Select a procedure and perform it</td>
<td>Design investigation for a specific purpose or research question</td>
<td>Initiate, design, and conduct a project that specifies a problem, identifies solution paths, solves the problem, and reports results</td>
</tr>
<tr>
<td></td>
<td>Calculate, measure, apply a rule (e.g., rounding)</td>
<td>Solve routine problem applying multiple concepts or decision points</td>
<td>Use reasoning, planning, and supporting evidence</td>
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<tr>
<td></td>
<td>Apply algorithm or formula</td>
<td>Retrieve information to solve a problem</td>
<td>Translate between problem &amp; symbolic notation when not a direct translation</td>
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<tr>
<td></td>
<td>Solve linear equations</td>
<td>Translate between representations</td>
<td></td>
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<tr>
<td></td>
<td>Make conversions</td>
<td></td>
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<tr>
<td>Depth of Thinking (Webb)+ Type of Thinking (Revised Bloom)</td>
<td>DOK Level 1 Recall &amp; Reproduction</td>
<td>DOK Level 2 Basic Skills &amp; Concepts</td>
<td>DOK Level 3 Strategic Thinking &amp; Reasoning</td>
<td>DOK Level 4 Extended Thinking</td>
</tr>
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<tr>
<td>Analyze</td>
<td>• Retrieve information from a table or graph to answer a question</td>
<td>• Categorize data, figures</td>
<td>• Compare information within or across data sets or texts</td>
<td>• Analyze multiple sources of evidence or data sets</td>
</tr>
<tr>
<td></td>
<td>• Identify a pattern/trend</td>
<td>• Organize, order data</td>
<td>• Analyze and draw conclusions from data, citing evidence</td>
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<tr>
<td></td>
<td></td>
<td>• Select appropriate graph and organize &amp; display data</td>
<td>• Generalize a pattern</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Interpret data from a simple graph</td>
<td>• Interpret data from complex graph</td>
<td></td>
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<tr>
<td></td>
<td></td>
<td>• Extend a pattern</td>
<td></td>
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<tr>
<td>Evaluate</td>
<td></td>
<td>Cite evidence and develop a logical argument</td>
<td>Apply understanding in a novel way, provide argument or justification for the new application</td>
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<tr>
<td></td>
<td></td>
<td>Compare/contrast solution methods</td>
<td></td>
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<td></td>
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<td>Verify reasonableness</td>
<td></td>
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<tr>
<td>Create</td>
<td>• Brainstorm ideas, concepts, problems, or perspectives related to a topic or concept</td>
<td>• Generate conjectures or hypotheses based on observations or prior knowledge and experience</td>
<td>• Develop an alternative solution</td>
<td>• Synthesize information across multiple sources or data sets</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Synthesize information within one data set</td>
<td>• Design a model to inform and solve a practical or abstract situation</td>
<td></td>
</tr>
</tbody>
</table>